CBSE Class 12 Physics

NCERT Solutions

Chapter – 1

Electric Charges and Fields

1: What is the force between two small charged spheres having charges of $2\times10^{-7}\mathrm{C}$ and $3\times10^{-7}\mathrm{C}$ placed 30 cm apart in air?

Ans: Repulsive force of magnitude $6\times10^{-3}N$

Charge on the first sphere, $q_{1=}2\times10^{-7}C$

Charge on the second sphere, $q_2=3\times10^{-7}C$

Distance between the spheres, r = 30 cm = 0.3 m

Electrostatic force between the spheres is given by the relation,

$F=\frac{kq_1q_2}{r^2}$

Where, k = $\frac{1}{4\pi\varepsilon_0}$

$\frac{1}{4\pi\epsilon_0}=9\times10^9\mathrm{Nm}^2C^2$

$F=\frac{9\times10^9\times2\times10^{-7}\times3\times10^{-7}}{(0.3)^2}=6\times10^{-3}N$

Hence, force between the two small charged spheres is $6\times10^{-3}N$. The charges are of same nature. Hence, force between them will be repulsive.

2: The electrostatic force on a small sphere of charge $0.4\mu C$ due to another small sphere of charge $-0.8\mu C$ in air is $0.2\mathrm{N}$ (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Ans:

a. Electrostatic force on the first sphere, F = $0.2\mathrm{N}$

   Charge on this sphere, q_1= $0.4\mu C=0.4\times10^{-6}C$

   Charge on the second sphere, q_2 = $-0.8\mu C=-0.8\times10^{-6}C$

   Electrostatic force between the spheres is given by the relation,

   F=$\frac{kq_1q_2}{r^2}$

   where, k= $\frac{1}{4\pi\varepsilon_0}$ and, ε_0 = Permittivity of free space

And, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^8 \mathrm{Nm}^{-2}\mathrm{C}^{-2}$
$r^2 = \frac{q_1 q_2}{4\pi\in_0 F}$
$= 144 \times 10^{-4}$
$r = \sqrt{144 \times 10^{-4}} = 0.12\mathrm{m}$
The distance between the two spheres is 0.12m.

b. Both the spheres attract each other with the same force. Therefore, the force on the second sphere due to the first is 0.2N.

3: Check that the ratio $\frac{ke^2}{Gm_r m_p}$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans: The given ratio is $\frac{ke^2}{Gm_e m_p}$.

Where,

G = Gravitational constant

Its unit is $\mathrm{Nm}^2\mathrm{kg}^{-2}$

m_e and m_p = Masses of electron and proton. Their unit is kg.

e = Electric charge. Its unit is C.

ε_0 = Permittivity of free space

Its unit is $\mathrm{N}^{-1}\mathrm{m}^{-2}\mathrm{C}^2$

Therefore, unit of the given ratio $\frac{ke^2}{Gm_c m_p}$

$\frac{[\mathrm{Nm}^2\mathrm{C}^{-2}][\mathrm{C}^2]}{[\mathrm{Nm}^2\mathrm{kg}^{-2}][\mathrm{kg}][\mathrm{kg}]}$

= $\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0$

Hence, the given ratio is dimensionless.

$e = 1.6 \times 10^{-19} C$
$G = 6.67 \times 10^{-11} \mathrm{Nm}^2\mathrm{kg}^{-2}$
$me = 9.1 \times 10^{-31} kg \quad mp = 1.66 \times 10^{-27} kg$

Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_e m_p} = \frac{(9\times10^9)(1.6\times10^{-19})^2}{(6.67\times10^{-11})(9.1\times10^{-31})(1.67\times10^{-27})} \approx 2.29 \times 10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

4:

a. **Explain the meaning of the statement 'electric charge of a body is quantized'.**
b. **Why can one ignore quantization of electric charge when dealing with macroscopic i.e.., large scale charges?**

Ans:

a. Electric charge of a body is quantized. It means charge transfer to a body in integral number (n=1,2,3.....). Charges are not transferred in fractions. Hence, a body possesses total charge only in integral multiples of base unit of charge i.e. q=$\pm$ne where e=1.6$\times$$10^{-19}$ C
b. In macroscopic or large scale charges, the charges used are huge as compared to the magnitude of the elementary electric charge. Hence, quantization of electric charge is of no use on macroscopic scale. Therefore, it is ignored and it is considered that electric charge is continuous.

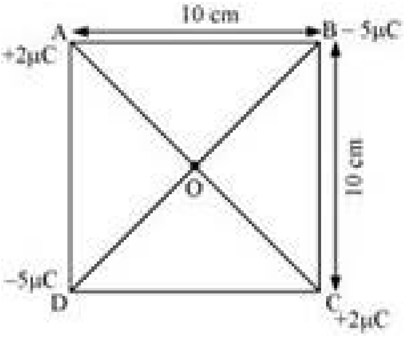
5: When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.
Ans: On rubbing glass rod with a silk cloth, heat is generated by friction and this generated heat is used to remove loosely bound electrons of glass rod and it acquires a positive charge and silk cloth will acquire negative charge due to transfer of electrons from glass rod to silk cloth.
Ans. This phenomenon of charging is called charging by friction. The net charge on the system of two rubbed bodies is zero before and after rubbing. Therefore in an isolated system, the total charge always remains constant. This is consistent with the law of conservation of energy.

---

6 : Four point charges qA = $2\mu_{\mathrm{C}}$ **, qB =** $-5\mu_{\mathrm{C}}$ **-,** $qC = 2\mu c$**, and** $qD = -5\mu_C$ **qD are located at the corners of a square ABCD of side 10 cm. What is the force on a**

charge of 1μC placed at the centre of the square?

Ans: The given figure shows a square of side 10 cm with four charges placed at its corners. O is the centre of the square.



Where,

(Sides) AB = BC = CD = AD = 10cm

(Diagonals) $AC = BD = 10\sqrt{2}\text{cm}$

$AO = OC = DO = OB = 5\sqrt{2}\text{cm}$

A charge of 1μC is placed at point O.

The charge at *O* experiences equal and opposite forces due to the charges placed at *A* and at *C* since they are of same nature. Hence, they cancel each other. So also, it experiences equal and opposite forces due to the charges at *B and D and consequently,* they cancel each other. Therefore, net force on the charge placed at O due to the four charges placed at the corner of the square is zero.

7:

a. **An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?**
b. **Explain why two field lines never cross each other at any point?**

Ans:

a. An electrostatic field line is a continuous curve starting at a positive charge and ending at a negative charge. Electric field in a region is always continuous. Therefore the field lines too are continuous and cannot have sudden breaks.
b. The tangent drawn to the electric field line at a point gives the direction of the electric field at that point. If two field lines cross each other at a point then there will be two tangents which represent the direction of Electric field at the same point which is not

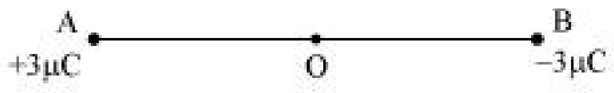
possible. Hence, two field lines never cross each other.

8 : Two point charges $q_A = 3\mu C$ and $q_B = -3\mu C$ are located 20 cm apart in vacuum.

a. **What is the electric field at the midpoint O of the line AB joining the two charges?**
b. **If a negative test charge of magnitude $1.5 \times 10^{-9}\mathrm{C}$ is placed at this point, what is the force experienced by the test charge?**

Ans:

a. The situation is represented in the given figure. O is the mid-point of line AB.
   Distance between the two charges, AB = 20 cm



$\therefore AO = OB = 10\mathrm{cm}$

Net electric field at point O = E

Electric field at point O caused by $+3\mu C$ charge,

$\vec{E}_1 = \frac{q_B}{4\pi\varepsilon_0 (OA)^2}\overrightarrow{OB} = \frac{(3\times10^{-6})(9\times10^{9})}{(10\times10^{-2})^2}\overrightarrow{OB}$

Where,

ε_0 = Permittivity of free space

$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\mathrm{Nm}^2 C^{-2}$

Magnitude of electric field at point O caused by $-3\mu C$ charge,

$\vec{E}_2 = \frac{q_B}{4\pi\varepsilon_0 (OB)^2}\overrightarrow{OB} = \frac{(3\times10^{-6})(9\times10^{9})}{(10\times10^{-2})^2}\overrightarrow{OB}$

$\vec{E} = \vec{E}_1 + \vec{E}_2$

Since $\left|\vec{E}_1\right| = \left|\vec{E}_2\right|$,

$\vec{E} = \frac{2(3\times10^{-6})(9\times10^{9})}{(10\times10^{-2})^2}\overrightarrow{OB} = 5.4 \times 10^6\mathrm{NC}^{-1}$

$= 5.4 \times 10^6 N/C$ along OB

Therefore, the electric field at mid-point O is $= 5.4 \times 10^6 N/C$ along OB

b. A test charge of amount $1.5 \times 10^{-9}\mathrm{C}$ is placed at mid-point O.

$q = 1.5 \times 10^{-9}\text{C}$

Force experienced by the test charge = F

$\therefore F = qE$

$= 1.5 \times 10^{-9} \times 5.4 \times 10^{6}$

$= 8.1 \times 10^{-3} N$

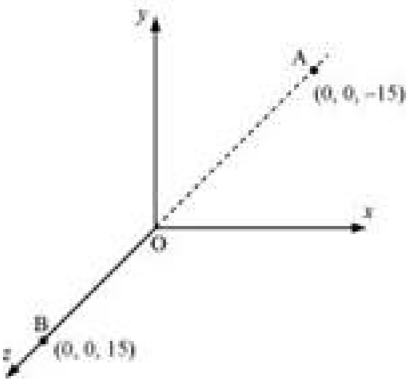
The force is directed along line OA. This is because the negative test charge is repelled by the charge placed at point B but attracted towards point A.

Therefore, the force experienced by the test charge is $8.1 \times 10^{-3} N$ along OA.

---

9 : A system has two charges $q_A = 2.5 \times 10^{-7}\text{C}$ and $qB = -2.5 \times 10^{-7}\text{C}$ located at points A: (0, 0, -15cm) and B: (0, 0, + 15cm), respectively. What are the total charge and electric dipole moment of the system?

Ans: Both the charges can be located in a coordinate frame of reference as shown in the given figure.



At A, amount of charge, $q_A = 2.5 \times 10^{-7} C$

At B, amount of charge, $q_B = -2.5 \times 10^{-7} C$

Total charge of the system,

q = $q_A + q_B$

q= $2.5 \times 10^{7} - 2.5 \times 10^{-7}$

= 0

Distance between two charges at points A and B,

d = 15 + 15 = 30cm = 0.3m

Electric dipole moment of the system is given by,

$p = q_A \times d = q_B \times d$

$= 2.5 \times 10^{-7} \times 0.3$

$= 7.5 \times 10^{-3}\text{Cm}$ along negative z-axis

Therefore, the electric dipole moment of the system is $7.5 \times 10^{-8}\mathrm{Cm}$ along the negative *z*-axis.

10: An electric dipole with dipole moment $4 \times 10^{-9}\mathrm{Cm}$ is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 NC^{-1}$ .Calculate the magnitude of the torque acting on the dipole.

Ans: Electric dipole moment, $p = 4 \times 10^{-9}\mathrm{Cm}$
Angle made by *p* with a uniform electric field, $\theta = 30^\circ$
Electric field, $E = 5 \times 10^4 NC^{-1}$
Torque acting on the dipole is given by the relation,
$\tau = pE\sin\theta$
$= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30$
$= 20 \times 10^{-5} \times \frac{1}{2}$
= 10^{-4} Nm
Therefore, the magnitude of the torque acting on the dipole is 10^{-4}N m.

11: A polythene piece rubbed with wool is found to have a negative charge of $3 \times 10^{-7}C$

a. **Estimate the number of electrons transferred (from which to which?)**
b. **Is there a transfer of mass from wool to polythene?**

Ans:

a. When polythene is rubbed against wool, a number of electrons get transferred from wool to polythene. Hence, wool becomes positively charged and polythene becomes negatively charged.
Amount of charge on the polythene piece, $q = -3 \times 10^{-7}C$
Amount of charge on an electron, $e = -1.6 \times 10^{-19}C$
Number of electrons transferred from wool to polythene = n
n can be calculated using the relation, q = ne
$n = \frac{q}{e}$
$= \frac{-3\times 10^{-7}}{-1.6\times 10^{-19}}$

$= 1.87 \times 10^{12}$

Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12}.

b. Yes.

There is a transfer of mass taking place. This is because an electron has mass,

$me = 9.1 \times 10^{-3} kg$

Total mass transferred to polythene from wool,

$m = me \times n$

$= 9.1 \times 10^{-31} \times 1.85 \times 10^{12}$

$= 1.706 \times 10^{-18} kg$

Hence, a negligible amount of mass is transferred from wool to polythene.

12:

a. **Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is $6.5 \times 10^{-7} C$? The radii of A and B are negligible compared to the distance of separation.**
b. **What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?**

Ans:

a. Charge on sphere A, $q_{A=}$ Charge on sphere B, q_B=6.5 $\times$ 10^{-7}C

Distance between the spheres, r = 50cm = 0.5m

Force of repulsion between the two spheres,

$F = \frac{q_A q_B}{4\pi \in_0 r^2}$

Where, ε_0 is the permittivity of free space

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \mathrm{Nm}^2\mathrm{C}^{-2}$

$F = \frac{9 \times 10^9 \times \left(6.5 \times 10^{-7}\right)^2}{(0.5)^2}$

$= 1.52 \times 10^{-2} N$

Therefore, the force between the two spheres is $= 1.52 \times 10^{-2} N$

b. After doubling the charge,

Charge on sphere A, q_A = Charge on sphere B q_B= 2 × 6.5 × 10^{-7}C =1.3 × 10^{-6}C

The distance between the spheres is halved.

$\therefore r=\frac{0.5}{2}=0.25\mathrm{m}$

Force of repulsion between the two spheres,

$F=\frac{q_Aq_B}{4\pi\in_0r^2}$

$=\frac{9\times10^9\times1.3\times10^{-5}\times1.3\times10^{-6}}{(0.25)^2}$

$=16\times1.52\times10^{-2}$

= 0.243 N

Therefore, the force between the two spheres is = 0.243 N

13: Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Ans: Distance between the spheres, A and B, r = 0.5m

Initially, the charge on each sphere, $q=6.5\times10^{-7}C$

When sphere A is touched with an uncharged sphere C, charge from A is transferred to sphere C. Since the spheres are identical, the charges are shared equally.Hence, charge on each of the spheres, A and C, is $\frac{q}{2}$.

When sphere C with charge $\frac{q}{2}$ is brought in contact with sphere B with charge q, total charges on the system will divide into two equal halves given as,

$\frac{\frac{q}{2}+q}{2}=\frac{3q}{4}$

Hence, charge on each of the spheres, C and B, is $\frac{3q}{4}$.

Force of repulsion between sphere A having charge $\frac{q}{2}$ and sphere B having charge $\frac{3q}{4}$ is

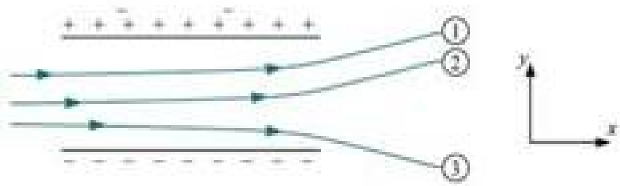
$\frac{\frac{q}{2}\times\frac{3q}{4}}{4\pi\in_0r^2}=\frac{3q^2}{8\times4\pi\in_0r^2}$

$=9\times10^9\times\frac{3\times\left(6.5\times10^{-7}\right)^2}{8\times(0.5)^2}$

$=5.703\times10^{-3}N$

Therefore, the force of attraction between the two spheres is $=5.703\times10^{-3}N$.

14: Figure shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Ans: Opposite charges attract each other and like charges repel each other. It can be observed that particles 1 and 2 both move towards the positively charged plate and move away from the negatively charged plate. Hence, these two particles are negatively charged. It can also be observed that particle 3 moves towards the negatively charged plate and moves away from the positively charged plate. Hence, particle 3 is positively charged.
The charge to mass ratio q/m is directly proportional to the displacement or amount of deflection for a given velocity. Since the deflection of particle 3 is the maximum, it has the highest charge to mass ratio.

15: Consider a uniform electric field $\mathbf{E} = 3 \times 10^3 \mathrm{iN/C}$. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes a 60^o angle with the *x*-axis?

Ans:

a. Electric field intensity, $\overline{E} = 3 \times 10^3 \mathrm{i} N/C$
Magnitude of electric field intensity, $|\overline{E}| = 3 \times 10^3 N/C$
Side of the square, s = 10 cm = 0.1 m
Area of the square, A =side^2 =0.01 m^2
The plane of the square is parallel to the *y*-*z* plane. Hence, angle between the unit vector normal to the plane and electric field, $\theta = 0^\circ$
Flux (Φ) through the plane is given by the relation,
$\Phi = |\overline{E}| A \cos\theta$
$= 3 \times 10^3 \times 0.01 \times \cos 0^\circ$
$= 30 \mathrm{Nm^2/C}$

b. Plane makes an angle of 60° with the x-axis. Hence, θ = 60°
Flux, $\Phi = |\overline{E}| A \cos\theta$
$= 3 \times 10^3 \times 0.01 \times \cos 60^\circ$
$= 30 \times \frac{1}{2}$
$= 15 Nm^2/C$

16: What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans: All the faces of a cube are parallel to the coordinate axes. In a uniform electric field, the number of field lines entering the cube is equal to the number of field lines exiting out of the cube. As a result, net flux through the cube is zero.

17: Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3 \mathrm{Nm}^2/\mathrm{C}$**.**

a. **What is the net charge inside the box?**
b. **If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?**

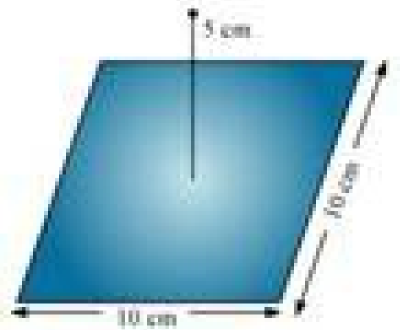
Ans:

a. Net outward flux through the surface of the box, $\Phi = 8.0 \times 10^3 \mathrm{Nm}^2/\mathrm{C}$
For a body containing net charge q, flux is given by the relation,
$\phi = \frac{q}{\epsilon_0}$
ϵ_0 = Permittivity of free space $= 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$
$q = \phi \varepsilon_0$
$= 8.854 \times 10^{-12} \times 8.0 \times 10^3$
$= 7.08 \times 10^{-8} \mathrm{C}$
$= 0.07 \mu C$
Therefore, the net charge inside the box is $0.07 \mu \mathrm{C}$.
b. No

Net flux piercing out through a body depends on the net charge contained in the body. If net flux is zero, then it can be inferred that net charge inside the body is zero. The body may have equal amount of positive and negative charges.

18: A point charge +10μC is a distance 5 cm directly above the center of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10 cm.)

Ans: The square can be considered as one face of a cube of edge 10 cm with a center where charge q is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.



$\phi_{\text{san}\,\alpha} = \frac{q}{\epsilon_0}$

Hence, electric flux through one face of the cube i.e.., through the square,

$\phi = \frac{\phi_{T\alpha ai}}{6} = \frac{1}{6}\frac{q}{\epsilon_0}$

Where,

$\in_0$ = Permittivity of free space

$= 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$

$q = 10^{\mu C} = 10 \times 10^{-6} C$

$\therefore \phi = \frac{1}{6} \times \frac{10 \times 10^{-6}}{8.854 \times 10^{-12}}$ = $1.88 \times 10^5 \mathrm{Nm}^2\mathrm{C}^{-1}$

Therefore, electric flux through the square is $1.88 \times 10^5 \mathrm{Nm}^2\mathrm{C}^{-1}$

19: A point charge of 2.0μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans: Net electric flux (Φ Net) through the cubic surface is given by,

$\phi_{\text{Net}} = \frac{q}{\epsilon_0}$

Where,
ε_0 = Permittivity of free space
$= 8.854 \times 10^{-12} N^{-1} C^2 \mathrm{m}^{-2}$
q = Net charge contained inside the cube $= 2.0\mu C = 2 \times 10^{-6} C$
$\therefore \phi_{\mathrm{N\varepsilon t}} = \frac{2 \times 10^{-6}}{8.854 \times 10^{-12}}$
$= 2.26 \times 10^5 Nm^2C^{-1}$
The net electric flux through the surface is $= 2.26 \times 10^5 Nm^2C^{-1}$

20: A point charge causes an electric flux of $-1.0 \times 10^3 \mathrm{Nm}^2/\mathrm{C}$ to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge.

a. **If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?**
b. **What is the value of the point charge?**

Ans:

a. Electric flux, $\Phi = -1.0 \times 10^3 Nm^2/C$
Radius of the Gaussian surface,
r = 10.0 cm
Electric flux piercing out through a surface depends on the net charge enclosed inside the Gaussian surface. The flux is independent of the size and shape of the Gaussian surface.
When the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same i.e $-1.0 \times 10^3 \mathrm{Nm}^2 \mathrm{C}^{-1}$
b. Electric flux is given by the relation,
$\phi = \frac{q}{\epsilon_0}$
Where,
q = Net charge enclosed by the spherical surface
ε_0 = Permittivity of free space = $8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$
$\therefore q = \phi \in_0$
$= -1.0 \times 10^3 \times 8.854 \times 10^{-12}$
$= -8.854 \times 10^{-9} C$
= - 8.854 nC
Therefore, the value of the point charge is 8.854 nC.

21: A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is $1.5 \times 10^3 \mathrm{N/C}$ and points radially inward, what is the net charge on the sphere?

Ans: The magnitude of the Electric field intensity (E) at a distance (d) from the centre of a sphere containing net charge *q* is given by the relation,

$E = \frac{q}{4\pi\epsilon_0 d^2}$

Where,

q = Net charge = $1.5 \times 10^3 \mathrm{N/C}$

d = Distance from the centre = 20 cm = 0.2 m

ε_0 = Permittivity of free space

And,

$\therefore q = E(4\pi\epsilon_0)d^2$

$= \frac{1.5 \times 10^3 \times (0.2)^2}{9 \times 10^9}$

$6.67 \times 10^{-9} = 6.67 \mathrm{nC}$

Since the electric field at the point is directed radially inwards, the charge contained by the sphere is negative. The net charge on the sphere is -6.67 nC

22: A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC /m^2.

a. **Find the charge on the sphere.**
b. **What is the total electric flux leaving the surface of the sphere?**

Ans:

a. Diameter of the sphere, *d* = 2.4 m
   Radius of the sphere, *r* = 1.2 m
   Surface charge density, σ = 80.0 $\mu C/m^2 = 80 \times 10^{-6} C/m^2$
   Total charge on the surface of the sphere,
   Q = Charge density × Surface area
   $= \sigma \times 4\pi r^2$
   $= 80 \times 10^{-5} \times 4 \times 3.14 \times (1.2)^2 = 1.447 \times 10^{-3}\mathrm{C}$

Therefore, the charge on the sphere is $1.447 \times 10^{-3}\mathrm{C}$.

b. Total electric flux ($\phi_{\text{ Total}}$) leaving out the surface of a sphere containing net charge Q is given by the relation,

$\phi_{\text{ Total }} = \frac{Q}{\epsilon_0}$

Where,

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$

$Q = 1.447 \times 10^{-3} C$

$\phi_{Total} = \frac{1.44 \times 10^{-3}}{8.854 \times 10^{-12}}$

$= 1.63 \times 10^3 NC^{-1} m^2$

Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^8 NC^{-1} m^2$.

23: An infinite line charge produces a field of $9 \times 10^4 N/C$ at a distance of 2 cm. Calculate the linear charge density.

Ans: Electric field produced by the infinite line charges at a distance d having linear charge density λ is given by the relation,

$E = \frac{\lambda}{2\pi\epsilon_0 d}$

$\lambda = 2\pi\varepsilon_0 dE$

Where,

d = 2 cm = 0.02 m

$E = 9 \times 10^4 N/C$

ϵ_0 = Permittivity of free space

$\lambda = \frac{4\pi\varepsilon_0 dE}{2}$

$= \frac{1}{9 \times 10^9} \frac{9 \times 10^4 \times 0.02}{2}$

$= 10^{-7}\mathrm{C/m}$

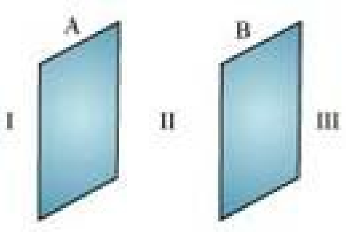
Therefore, the linear charge density is $10^{-7}\mathrm{C/m}$

24: Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22}\mathrm{C/m}^2$. What is E:

a. **in the outer region of the first plate,**

b. **in the outer region of the second plate, and (c) between the plates?**

Ans: The situation is represented in the following figure.



A and B are two parallel plates close to each other. Outer region of plate A is labelled as **I**, outer region of plate B is labelled as **III**, and the region between the plates, A and B, is labelled as **II.**

Charge density of plate A, $\sigma = 17.0 \times 10^{-22} \mathrm{C/m^2}$

Charge density of plate B, $\sigma = -17.0 \times 10^{-2} \mathrm{C/m^2}$

Electric field intensity at any point due to an infinitely large plane charged sheet is independent of the distance of the point from the sheet.

(a) At any point in the outer region **(I)** of plate *A*, the field intensity due to A is opposite to the field intensity due to B, since the charge densities on the plates are of opposite signs..

Therefore the field intensity is given by E_A- E_B.

$\left|\vec{E}_A\right| = \left|\vec{E}_B\right| = \frac{\sigma}{2\varepsilon_0}$

$E_A - E_B = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

(b) Similarly, at any point in the outer region **(III)**of the plate *B* is also zero.

(c) In the region **II,** the electric field intensities due to both the plates act in the same direction. Electric field *E* in region **II** is given by the relation,

Where, ε_0 =Permittivity of free space $= 8.854 \times 10^{-12} N^{-1} C^2 m^{-2}$

$\therefore E = \frac{17.0 \times 10^{-22}}{8.854 \times 10^{-12}}$

$= 1.92 \times 10^{-10} \mathrm{N/C}$

Therefore, electric field between the plates is $= 1.92 \times 10^{-10} \mathrm{N/C}$

25: An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55 \times 10^4 \mathrm{NC}^{-1}$ in Millikan's oil drop experiment. The density of the oil is 1.26 g cm-^{3}. Estimate the radius of the drop.

$\left(g = 9.81\text{ms}^{-2}; e = 1.60 \times 10^{-19}\text{C}\right)$.

Ans: Excess electrons on an oil drop, *n* = 12

Electric field intensity, $E = 2.55 \times 10^4 \text{NC}^{-1}$

ρ = density of foil $= 1.26 \times 10^3 \quad kg/m^3$

Acceleration due to gravity, g = 9.81ms^{-2}

Charge on an electron, $e = 1.6 \times 10^{-19} C$

Radius of the oil drop = *r*

At equilibrium,Force (*F*) due to electric field *E* is equal to the weight of the oil drop (*W*)

F = W

qE = *mg*

Since, q = Net charge on the oil drop = - ne, and

m = Mass of the oil drop

= Volume of the oil drop × Density of oil

$mg = \frac{4}{3}\pi r^3 \rho g$

$E(ne) = \frac{4}{3}\pi r^3 \rho g$

$r = \sqrt[3]{\frac{3Ene}{4\pi\rho g}}$

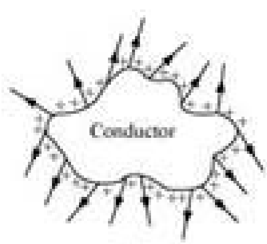
$= \sqrt[3]{\frac{3\left(2.55\times10^4\right)(12)\left(1.60\times10^{-19}\right)}{4(3.14)\left(1.26\times10^3\right)(9.81)}}$

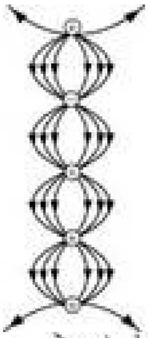
$= 9.82 \times 10^{-7}\text{m}$

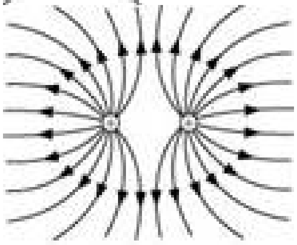
$= 9.82 \times 10^{-4}\text{mm}$

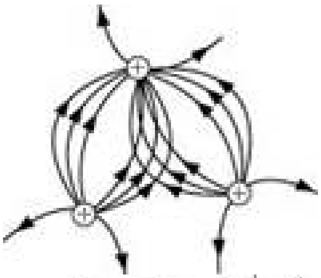
Therefore, the radius of the oil drop is $= 9.82 \times 10^{-4}\text{mm}$

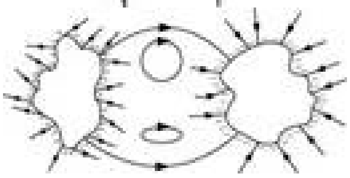
26: Which among the curves shown in Fig. cannot possibly represent electrostatic field lines?

a. 

b. 

c. 

d. 

e. 

Ans:

a. The field lines showed in (a) do not represent electrostatic field lines because field lines must be normal to the surface of the conductor.
b. The field lines showed in (b) do not represent electrostatic field lines because the field lines cannot emerge from a negative charge and cannot terminate at a positive charge.
c. The field lines showed in (c) represent electrostatic field lines. This is because the field lines emerge from the positive charges and repel each other.
d. The field lines showed in (d) do not represent electrostatic field lines because the field lines do not intersect each other.
e. The field lines showed in (e) do not represent electrostatic field lines because electric field lines do not form closed loops.

27: In a certain region of space, electric field is along the z-direction

throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive *z*-direction, at the rate of 10^5 N C^{-1} per meter. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative *z*-direction?

Ans: Dipole moment of the system, p = q $\times$ dl = -10^{-7}Cm. Rate of increase of electric field per unit length,

$\frac{dE}{dl} = 10^5 \mathrm{NC}^{-1}\mathrm{m}^{-1}$

Force (F) experienced by the system is given by the relation,

F = qE

$F = q\frac{dE}{dl} \times dl$

$= p \times \frac{dE}{dl}$

$= -10^{-7} \times 10^{-5}$

$= -10^{-2}N$

The force is $-10^{-2}N$ in the negative z-direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180°.

Torque τ is given by the relation,

$\tau = pE\sin\theta = pE\sin 180 = 0$

Therefore, the torque experienced by the system is zero.

28:

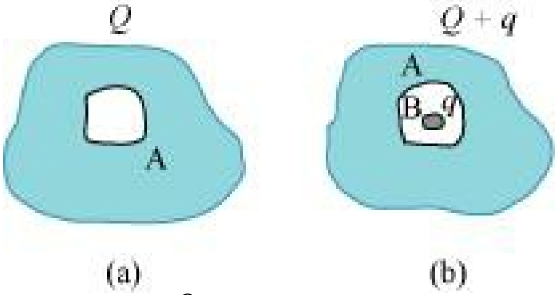
a. **A conductor A with a cavity as shown in Fig. 1.36(a) is given a charge *Q*. Show that the entire charge must appear on the outer surface of the conductor.**
b. **Another conductor B with charge *q* is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is *Q* + *q* [Fig. 1.36(b)].**
c. **A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.**

Ans:

a. Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity E inside the charged conductor is zero.

Let q is the charge inside the conductor and ε_0 is the permittivity of free space.

According to Gauss's law,



Flux, $\phi = \frac{Q}{\varepsilon_0}$

Since, inside the conductor, E = 0,Q =0

Therefore, charge inside the conductor is zero.

The entire charge Q appears on the outer surface of the conductor.

b. The outer surface of conductor A has a charge of amount Q. Another conductor B having charge +q is kept inside conductor A and it is insulated from A. Hence, a charge of amount - q will be induced in the inner surface of conductor A and +q is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is Q + q.

c. A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

29: A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left[\frac{\sigma}{2\in_0}\right]\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

Ans: Let us consider a conductor with a cavity or a hole. Electric field inside the cavity is zero.

Let E is the electric field just outside the conductor, q is the electric charge, σ is the charge density, and $\in_0$ is the permittivity of free space.

According to Gauss's law,$\phi = \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}\phi = \frac{Q}{\varepsilon_0}$

$\vec{E} \cdot d\vec{S} = \frac{\sigma dS}{\varepsilon_0}\hat{n}$

$\therefore E = \frac{\sigma}{\epsilon_0}\hat{n}$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\varepsilon_0}\hat{n}$. This field is a superposition of field due to the cavity E_1 and the field due to the rest of the charged conductor E_2. Inside the conductor, since the total electric field is zero,these fields are equal and opposite.

$\vec{E} = \vec{E}_1 - \vec{E}_2 = 0$

$\left|\vec{E}_1\right| = \left|\vec{E}_2\right|$

But outside the conductor, they have equal magnitudes and act along the same direction.

$\vec{E}_1 + \vec{E}_2 = \vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

$\vec{E}_1 = \frac{\vec{E}}{2} = \frac{\sigma}{2\varepsilon_0}\hat{n}$

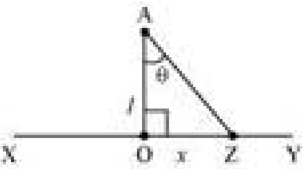
Therefore, the field due to the rest of the conductor is $\frac{\sigma}{2\varepsilon_0}\hat{n}$.

Hence, proved.

30: Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [*Hint:* Use Coulomb's law directly and evaluate the necessary integral.]

Ans: Take a long thin wire XY (as shown in the figure) of uniform linear charge density λ.

Consider a point A at a perpendicular distance *l* from the mid-point O of the wire, as shown in the following figure.



Let *E* be the electric field at point A due to the wire, XY.

Consider a small length element *dx* on the wire section with OZ = *x*

Let *q* be the charge on this piece.

$\therefore q = \lambda dx$

Electric field due to the piece,

$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(AZ)^2}$

Since $AZ = \sqrt{l^2 + x^2}$

$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{\left(l^2+x^2\right)}$

The electric field is resolved into two rectangular components. $dE\cos\theta$ is the perpendicular component and $dE\sin\theta$ is the parallel component.

When the whole wire is considered, the component $dE\sin\theta$ is cancelled.

Only the perpendicular component $dE\sin\theta$ affects point A.

Hence, effective electric field at point A due to the element dx is dE_1.

$dE_1 = \frac{\lambda dx\cos\theta}{4\pi\varepsilon_0\left(x^2+l^2\right)}$

In ΔAZO

$\tan\theta = \frac{x}{l}$

$x = l\tan\theta$ ...... (2)

On differentiating equation (2), we obtain

$\frac{dx}{d\theta} = l\sin^2\theta$

$dx = l\sin^2 d\theta$ ...... (3)

From equation (2),

$x^2 + l^2 = l^2 + \tan^2\theta$

$\therefore l^2\left(1+\tan^2\theta\right) = l^2\sec^2\theta$

$x^2 + l^2 = l^2\sin^2$ ....(4)

Putting equations (3) and (4) in equation (1), we obtain

$\therefore dE_1 = \frac{\lambda l\sec^2 d\theta}{4\pi\epsilon_0 l^2\sec^2\theta} \times \cos\theta$

$\therefore dE_1 = \frac{\lambda\cos\theta d\theta}{4\pi\in_0 l}$ .... (5)

For an infinitely long charged wire, the angle θ extends from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

By integrating equation (5), we obtain the value of field E_1 as, $E_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda}{4\pi\varepsilon_0}\cos\theta d\theta$

$E_1 = \frac{\lambda}{4\pi\epsilon_0 l} \times 2$

$E_1 = \frac{\lambda}{2\pi\in_0 l}$

Therefore, the electric field due to long wire is $\frac{\lambda}{2\pi\epsilon_0 l}$

31: It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge $\left(+\frac{1}{2}\right)$ e, and the 'down' quark (denoted by d) of charge (-1/3) e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Ans: A proton has three quarks. Let there be n up quarks in a proton, each having a charge of $+\left(\frac{2}{3}e\right)$.

Charge due to n up quarks $=\left(\frac{2}{3}e\right)n$

Number of down quarks in a proton = 3 – n

Each down quark has a charge of $-\frac{1}{3}e$.

Charge due to (3 – n) down quarks $=\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a proton = + e

$\therefore e=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$

$e=\left(\frac{2ne}{3}\right)-e+\frac{ne}{3}$

2e = ne

n = 2

Number of up quarks in a proton, n = 2

Number of down quarks in a proton = 3 – n = 3 – 2 = 1

Therefore, a proton can be represented as 'uud'.

A neutron also has three quarks. Let there be n up quarks in a neutron, each having a charge of $+\frac{2}{3}e$.

Charge on a neutron due to n up quarks $=\left(+\frac{3}{2}e\right)n$

Number of down quarks is 3 - n, each having a charge of $=\left(-\frac{3}{2}\right)e$.

Charge on a neutron due to (3 - n)down quarks = $\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a neutron = 0

$0=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$

$0=\frac{2}{3}en-e+\frac{ne}{3}$

e = ne, n = 1

Number of up quarks in a neutron, n = 1

Number of down quarks in a neutron = 3 - n = 2

Therefore, a neutron can be represented as 'udd'.

32:

a. **Consider an arbitrary electrostatic field configuration. A small test charge is**

placed at a null point (i.e., where E = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

b. **Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.**

Ans:

a. Let the equilibrium of the test charge be stable. If a test charge is in equilibrium and displaced from its position in any direction, then it experiences a restoring force towards a null point, where the electric field is zero. All the field lines near the null point are directed inwards towards the null point. There is a net inward flux of electric field through a closed surface around the null point. According to Gauss's law, the flux of electric field through a surface, which is not enclosing any charge, is zero. Hence, the equilibrium of the test charge can be stable.
b. Two charges of same magnitude and same sign are placed at a certain distance. The mid-point of the joining line of the charges is the null point. When a test charged is displaced along the line, it experiences a restoring force. If it is displaced normal to the joining line, then the net force takes it away from the null point. Hence, the charge is unstable because stability of equilibrium requires restoring force in all directions.

33: A particle of mass m and charge (- q) enters the region between the two charged plates initially moving along x-axis with speed vx (like particle 1 in Fig. 1.33). The length of plate is L and an uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m^{v2_s})$. Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Ans: Charge on a particle of mass m = - q

Velocity of the particle = vx

Length of the plates =L

Magnitude of the uniform electric field between the plates = E

Mechanical force, F = Mass (m) × Acceleration (a)

$a = \frac{F}{m}$

However, electric force, F = qE

Therefore, acceleration, $a = \frac{qE}{m}$

Time taken by the particle to cross the field of length L is given by,

$t = \frac{\text{Length of the plate}}{\text{speed of the charge}} = \frac{L}{v_x}$

In the vertical direction, initial velocity, u = 0

According to the third equation of motion, vertical deflection s of the particle can be obtained as,

$s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$

$s = \frac{qEL^2}{2mv_x^2}$

Hence, vertical deflection of the particle at the far edge of the plate is $qEL^2/\left(2mv_x^2\right)$. This is similar to the motion of horizontal projectiles under gravity.

34: Suppose that the particle in Exercise in 1.33 is an electron projected with velocity $v_x = 2.0\times10^6$ m s^{-1}. If E between the plates separated by 0.5 cm is $9.1 \times 10^2 \mathrm{N/C}$, where will the electron strike the upper plate? (| e | = $1.6 \times 10^{-19}\mathrm{C}, me = 9.1 \times 10^{-31}\mathrm{kg}$.)

Ans: Velocity of the particle, $v_x = 2.0 \times 10^6 \mathrm{m/s}$

Separation of the two plates, d = 0.5 cm = 0.005 m

Electric field between the two plates, $E = 9.1 \times 10^2 N/C$

Charge on an electron, $q = 1.6 \times 10^{-19} C$

Mass of an electron, me = $9.1 \times 10^{-31} kg$

Let the electron strike the upper plate at the end of plate L, when deflection is s.

Therefore,

$s = \frac{qEL^2}{2mv_x^2}$

$L = \sqrt{\frac{2mv_x^2 s}{qE}} = \sqrt{\frac{2\left(9.1\times10^{31}\right)\left(2.0\times10^6\right)^2\left(0.5\times10^{-2}\right)}{\left(1.6\times10^{-19}\right)\left(9.1\times10^2\right)}}$

$= \sqrt{\frac{2}{1.6}\times10^{-4}}$

$= 1.12 \times 10^{-2}\mathrm{m}$

Therefore, the electron will strike the upper plate after travelling 1.12cm.

CBSE Class 12 Physics

NCERT Solutions

Chapter - 2

Electrostatic Potential and Capacitance

1: Two charges $5 \times 10^{-8} C$ and $-3 \times 10^{-8} C$ are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

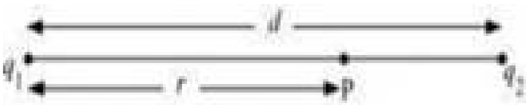
Ans: There are two charges,

$q_1 = 5 \times 10^{-8} C$

$q_2 = -3 \times 10^{-8} C$

Distance between the two charges, d = 16 cm = 0.16 m

Consider a point P between the two charges,on the line joining the two charges, as shown in the given figure.



r = Distance of point P from charge q_1

Let the electric potential (V) at point P be zero.

Potential at point P is the sum of potentials due to the fields of charges q_1 and q_2 respectively.

$V = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 (d-r)}$ ....(i)

Where,

$\in_0$ = Permittivity of free space

For V = 0, equation (i) reduces to

$\frac{q_1}{4\pi\varepsilon_0 r} = -\frac{q_2}{4\pi\varepsilon_0 (d-r)}$

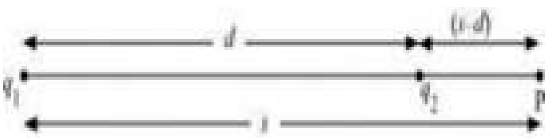
$\frac{q_1}{r} = \frac{-q_2}{d-r}$

$\frac{5\times10^{-3}}{r} = -\frac{(-3\times10^{-3})}{(0.16-r)}$

$\frac{0.16}{r} = \frac{8}{5}$

$\therefore r = 0.1m = 10\text{cm}$

Therefore, the the potential is zero at a point located between the two charges at a distance of 10 cm from the positive charge .



Let point P, where potential is zero, be outside the system of two charges at a distance ***s*** from the positive charge, as shown in the following figure.

For this arrangement, potential is given by,

$V = \frac{q_1}{4\pi\epsilon_0 s} + \frac{q_2}{4\pi\epsilon_0(s-d)}$ ...... (ii)

For V = 0, equation (ii) reduces to

$\Rightarrow \frac{q_1}{4\pi\epsilon_0 s} = -\frac{q_2}{4\pi\epsilon_0(s-d)}$

$\Rightarrow \frac{q_1}{s} = \frac{-q_2}{s-d}$

$\Rightarrow \frac{5\times10^{-3}}{s} = -\frac{(-3\times10^{-8})}{(s-0.16)}$

$\Rightarrow 1 - \frac{0.16}{s} = \frac{3}{5}$

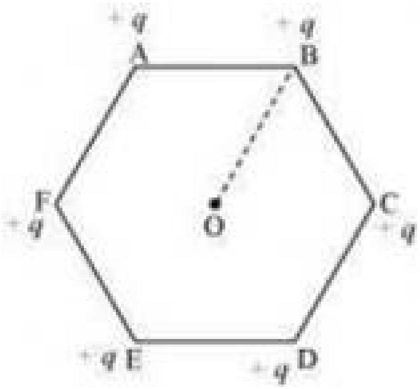
$\Rightarrow \frac{0.16}{s} = \frac{2}{5}$

$\therefore s = 0.4m = 40\text{cm}$

Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

2. A regular hexagon of side 10 cm has a charge 5μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Ans. The given figure shows six equal amount of charges, q, at the vertices of a regular hexagon.



Where,

Charge, $q = 5\mu C = 5 \times 10^{-6} C$

Side of the hexagon, *d* = AB = BC = CD = DE = EF = FA = 10 cm

For a regular hexagon, the distance of each vertex from center O, d = 10 cm

Electric potential at point O,

$V = \frac{kq}{r}$
Where, $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$
$\in_0$ = Permittivity of free space
$\therefore V = \frac{6 \times 9 \times 10^9 \times 5 \times 10^{-6}}{0.1} = 2.7 \times 10^6 V$
Therefore, the potential at the centre of the hexagon is $2.7 \times 10^6 V$.

3: Two charges $2\mu\text{C}$ and $-2\mu\text{C}$ are placed at points A and B 6 cm apart.

i. **Identify an equipotential surface of the system.**
ii. **What is the direction of the electric field at every point on this surface?**

Ans: The situation is represented in the given figure.

i. A surface at which potentially remains same at each point is known as equipotential surface. The equipotential surface selected may be a plane located at the midpoint of AB and normal to line joining the charges. On this plane, the potential is zero since all points on the plane are equidistant from the two charges.
ii. The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

4: A spherical conductor of radius 12 cm has a charge of $1.6 \times 10^{-7}\text{C}$ distributed uniformly on its surface. What is the electric field

i. **Inside the sphere**
ii. **Just outside the sphere**
iii. **At a point 18 cm from the centre of the sphere?**

Ans: Radius of the spherical conductor, r = 12 cm = 0.12 m
Charge q is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7} C$

i. Electric field inside a spherical metallic conductor is zero. The charge inside the sphere is zero and all the charge resides on the surface of the charged conductor.

ii. Electric field E just outside the conductor is given by the relation,
$E=\frac{kq}{r^2}$
Where, $k=\frac{1}{4\pi\varepsilon_0}$ and $\in_0$ = Permittivity of free space
$\Rightarrow \frac{1}{4\pi\epsilon_0}=9\times10^9\mathrm{Nm}^2C^{-c}$
$\therefore E=\frac{1.6\times10^{-7}\times9\times10^{-9}}{(0.12)^2}$

= 10^5 NC^{-1}

Therefore, the electric field just outside the sphere is 10^5 NC^{-1}.

iii. Electric field at a point 18 m from the centre of the sphere = E_1

Distance of the point from the centre, d = 18 cm = 0.18 m
$E_1=\frac{q}{4\pi\epsilon_0 d^2}$
$=\frac{(9\times10^9)(1.6\times10^{-7})}{(18\times10^{-2})^2}$
$=4.4\times10^4\mathrm{N/C}$
Therefore, the electric field at a point 18 cm from the centre of the sphere is
$4.4\times10^4\mathrm{N/C}$.

5: A parallel plate capacitor with air between the plates has a capacitance of 8 pF $(1pF=10^{-12}F)$. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Ans: Capacitance between the parallel plates of the capacitor, C = 8 pF

Initially, distance between the parallel plates was *d* and it was filled with air. Dielectric constant of air, k = 1

Capacitance, C, is given by the formula, $C=\frac{\varepsilon_0 A}{d}$
Where,
A = Area of each plate
$\in_0$ = Permittivity of free space
If distance between the plates is reduced to half, then new distance,$d_1=\frac{d}{2}$
Dielectric constant of the substance filled in between the plates, *k* = *6*
Hence, capacitance of the capacitor becomes
$C_1=\frac{\varepsilon_0 kA}{d_1}$

$= \frac{\varepsilon_0 kA}{\frac{d}{2}}$
$= 12\frac{\varepsilon_0 A}{d}$ (Since k =6)
$= 12 \times 8 = 96pF$
Therefore, the capacitance between the plates is 96 pF.

6: Three capacitors each of capacitance 9 pF are connected in series.

i. **What is the total capacitance of the combination?**
ii. **What is the potential difference across each capacitor if the combination is connected to a 120 V supply?**

Ans:

i. Capacitance of each of the three capacitors, C = 9 pF
Equivalent capacitance (C') of the series combination of the capacitors is given by the relation,
$\frac{1}{C^1} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$
$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}$
$\therefore C' = 3\mu F$
Therefore, total capacitance of the combination is $3\mu F$.
ii. Supply voltage, V = 100 V
Potential difference (*V*) across each capacitor is equal to one-third of the supply voltage.
$\therefore V' = \frac{V}{3} = \frac{120}{3} = 40V$
Therefore, the potential difference across each capacitor is 40 V.

7: Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

i. **What is the total capacitance of the combination?**
ii. **Determine the charge on each capacitor if the combination is connected to a 100 V supply.**

Ans:

i. Capacitances of the given capacitors a
C_1 = 2pF, C_2 = 3pF, C_3 = 4pF
For the parallel combination of the capacitors, equivalent capacitor *C* is given by the algebraic sum,
C = C_1 + C_2 + C_3
C = 2 + 3 + 4 = 9pF
Therefore, total capacitance of the combination is 9 pF.
ii. Supply voltage, V = 100 V
The voltage across all the three capacitors is same = V = 100 V
Charge on a capacitor of capacitance C and potential difference V is given by the relation,
q = CV
For C_1 = 2 pF,$q_1 = C_1V = 2 \times 100 = 200\text{pC}$
For C_2 = 3 pF, $q_2 = C_2V = 3 \times 100 = 300\text{pC}$
For C_3 = 4 pF,$q_3 = C_3V = 4 \times 100 = 400\text{pC}$

8: In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Ans: Area of each plate of the parallel plate capacitor, $A = 6 \times 10^{-3}m^2$
Distance between the plates, d = 3 mm $= 3 \times 10^{-3}\text{m}$
Supply voltage, V = 100 V
Capacitance C of a parallel plate capacitor is given by,
$C = \frac{\epsilon_0 A}{d}$
Where,
$\in_0$ = Permittivity of free space
$= 8.854 \times 10^{-12}N^{-1}m^{-2}C^{-2}$
$\therefore C = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$
$= 17.71 \times 10^{-12}F$
= 17.71pF
Potential V is related with the charge q and capacitance C as

$V=\frac{q}{C}$
$\therefore$ q = V C
$=1.771\times10^{-9}C$
$=100\times17.71\times10^{-12}$
Therefore, capacitance of the capacitor is 17.71pF and
the charge on each plate is 1.771×10^{-9}C.

9: Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

i. **While the voltage supply remained connected.**
ii. **After the supply was disconnected.**

Ans: Dielectric constant of the mica sheet, k = 6

i. Initial capacitance, $C=1.771\times10^{-11}$F
New capacitance, $C^r=kC=6\times1.771\times10^{-11}=106pF$
Supply voltage, V = 100 V
When the voltage supply remains connected and a dielectric is inserted between the plates of the capacitor, the potential difference across the plates remains constant and additional charge flows from the supply on to the plates so as to increase the capacitance.
New charge $q^1=C^1V=6\times1.7717\times10^{-9}=1.06\times10^{-8}C$
Potential across the plates remains 100 V.
ii. Dielectric constant, k = 6
Initial capacitance, $C=1.771\times10^{-11}F$
New capacitance $C^1=kC=6\times1.771\times10^{-11}=106pF$
If supply voltage is removed, and the dielectric is introduced, the capacitance of the capacitor increases,the charge on the plates remains constant, while the potential difference across the plates falls.
Charge $=1.771\times10^{-8}C$
Potential across the plates is given by,
$\therefore V'=\frac{q}{C^1}$
$=\frac{1.771\times10^{-9}}{106\times10^{-12}}$

= 16.7 V

10: A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Ans: Capacitance of the capacitor C = 12pF, $= 12 \times 10^{-12} F$

Potential difference, V = 50 V

Electrostatic energy stored in the capacitor is given by the relation,

$$E = \frac{1}{2} C V^2$$
$$= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$$
$$= 1.5 \times 10^{-8} J$$

Therefore, the electrostatic energy stored in the capacitor is $1.5 \times 10^{-8} J$.

11. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Ans: Capacitance of the capacitor, C = 600 pF ;Potential difference, V = 200 V

Electrostatic energy stored in the capacitor is given by,

$$U = \frac{1}{2} C_1 V^2$$
$$= \frac{1}{2} \times \left(600 \times 10^{-12}\right) \times (200)^2$$
$$= 1.2 \times 10^{-5} J$$

The charge Q on the capacitor is given by,

$$Q = C_1 V = 600 \times 10^{-12} \times 200 = 12 \times 10^{-8} \mathrm{C}$$

If supply is disconnected from the capacitor and another capacitor of capacitance C = 600 pF is connected to it, the charge Q on C_1 is shared between the two capacitors. The equivalent capacitance C' of the combination is given by,

$C' = C_1 + C_2 = 600 + 600 = 1200$ pF

The new potential V' across the capacitors is given by,

$$V' = \frac{Q}{C_1 + C_2}$$
$$= \frac{12 \times 10^{-8}}{12 \times 10^{-10}} = 100 \mathrm{V}$$

New electrostatic energy can be calculated as

$$U' = \frac{1}{2} C' V'^2 = \frac{1}{2} \left(12 \times 10^{-10}\right) (100)^2 = 6 \times 10^{-6} \mathrm{J}$$

Loss in electrostatic energy $= U - U'$

$= 12 \times 10^{-6} - 6 \times 10^{-6} = 6 \times 10^{-6}\text{J}$

Therefore, the electrostatic energy lost in the process is $6 \times 10^{-6}\text{J}$.

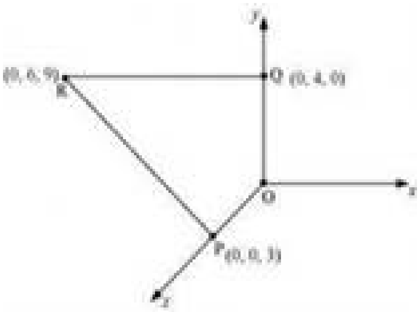
12: A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of $-2 \times 10^{-9}C$ from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Ans: Charge located at the origin, q = 8 mC $= 8 \times 10^{-3}C$

Magnitude of a small charge, which is taken from a point P to point R to point Q, q_1 =

$-2 \times 10^{-9}C$

All the points are represented in the given figure.



Point P is at a distance, d_1 = 3 cm, from the origin along z-axis. Point Q is at a distance, d_2 = 4 cm, from the origin along y-axis.

Potential at point P,

$V_1 = \frac{q}{4\pi\epsilon_0 \times d_2}$

Potential at point Q,

$V_2 = \frac{q}{4\pi\epsilon_0 \times d_2}$

Work done (W) by the electrostatic force is independent of the path.

$\therefore W = q_1 [V_2 - V_1]$

$= q_1 \left[\frac{q}{4\pi\epsilon_0 d_2} - \frac{q}{4\pi\epsilon_0 d_2}\right]$

$= \frac{qq_1}{4\pi\epsilon_0}\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$ ....(i)

where, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{Nm}^2 C^{-2}$

$\therefore W = 9 \times 10^9 \times 8 \times 10^{-3} \times \left(-2 \times 10^{-9}\right)\left[\frac{1}{0.04} - \frac{1}{0.03}\right]$

$= -144 \times 10^{-3} \times \left(\frac{-25}{3}\right)$

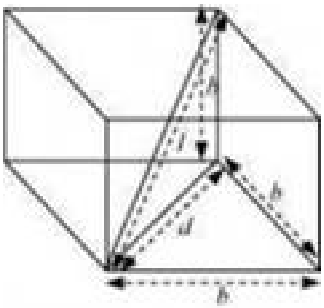
= 1.27J

Therefore, work done during the process is 1.27 J.

13: A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Ans: Length of the side of a cube = b

Charge at each of its vertices = q



A cube of side b is shown in the following figure.

d = Diagonal of one of the six faces of the cube

$d^2 = \sqrt{b^2 + b^2} = \sqrt{2b^2}$

$d = b\sqrt{2}$

l = Length of the diagonal of the cube

$l^2 = \sqrt{d^2 + b^2}$

$= \sqrt{(\sqrt{2}b)^2 + b^2}$

$= \sqrt{2b^2 + b^2} = \sqrt{3b^2}$

$l = b\sqrt{3}$

$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$

Is the difference between the centre of the cube and one of the eight vertices

The electric potential (V) at the centre of the cube is due to the presence of eight charges at the vertices.

$V = \frac{8q}{4\pi\varepsilon_0 r}$

$= \frac{8q}{4\pi\epsilon_0\left(b\frac{\sqrt{3}}{2}\right)}$

$= \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$

$\frac{4q}{\sqrt{3}\pi\epsilon_0 b}$

Therefore, the potential at the centre of the cube is $\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$

The electric field at the centre of the cube, due to the eight charges, gets cancelled. This is because the charges are distributed symmetrically with respect to the centre of the cube. Hence, the electric field is zero at the centre.

14: Two tiny spheres carrying charges $1.5\mu\text{C} \text{and} 2.5\mu\text{C}$ are located 30 cm apart. Find the potential and electric field:

i. **at the mid-point of the line joining the two charges, and**
ii. **at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.**

Ans: Two charges placed at points A and B are represented in the given figure. O is the mid-point of the line joining the two charges.

Magnitude of charge located at A, q_1 = $1.5\mu\text{C}$

Magnitude of charge located at B, q_2 =$2.5\mu\text{C}$

Distance between the two charges, d = 30 cm = 0.3 m

i. Let V_1 and E_1 are the electric potential and electric field respectively at O.

   V_1 = Potential due to charge at A + Potential due to charge at B$V_1 = \frac{q_1}{4\pi\in_0\left(\frac{d}{2}\right)} + \frac{q_2}{4\pi\epsilon_0\left(\frac{d}{2}\right)}$

   $= \frac{1}{4\pi\in_0\left(\frac{d}{2}\right)}(q_1 + q_2)$

   Where,

   $\in_0$ = Permittivity of free space

   $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 NC^2 m^{-2}$

   $V_1 = \frac{\left(9\times10^9\right)\left(10^{-6}\right)}{\left(\frac{0.30}{2}\right)}(2.5 + 1.5)$

   $= 2.4 \times 10^5 V$

   E_1 = Electric field due to q_2 - Electric field due to q_1

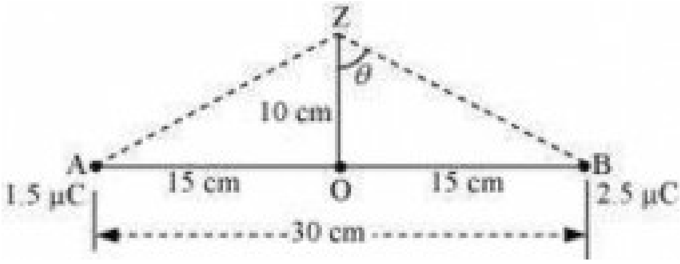
   $E = E_2 - E_1$

   $= \frac{q_2}{4\pi\in_0\left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi\epsilon_0\left(\frac{d}{2}\right)^2}$

$= \frac{(9\times10^{9})(10^{-6})}{\left(\frac{0.30}{2}\right)^{2}}(2.5-1.5)$

$= 4\times10^{5}\mathrm{Vm}^{-1}$

Therefore, the potential at mid-point is $2.4\times10^{5}V$ and the electric field at mid-point is $4\times10^{5}Vm^{-1}$. The field is directed from the larger charge to the smaller charge.



ii. Consider a point Z such that normal distance OZ = 10 cm = 0.1 m, as shown in the following figure.

V_2 and E_2 are the electric potential and electric field respectively at Z. It can be observed from the figure that distance,

BZ=AZ=$\sqrt{(0.1)^{2}+(0.15)^{2}}=0.18m$

V_2= Electric potential due to A + Electric Potential due to B

$= \frac{q_1}{4\pi\epsilon_0(AZ)}+\frac{q_1}{4\pi\epsilon_0(BZ)}$

$= \frac{9\times10^{9}\times10^{-5}}{0.18}(1.5+2.5)$

$= 2\times10^{5}V$

Electric field due to q at Z,

$E_A=\frac{q_1}{4\pi\varepsilon_0(AZ)^{2}}$

$= \frac{(9\times10^{9})(1.5\times10^{-6})}{(0.18)^{2}}=4.17\times10^{5}\mathrm{V/m}$

This field acts along AZ

Electric field due to q_2 at Z,

$E_B=\frac{q_2}{4\pi\varepsilon_0(BZ)^{2}}$

$= \frac{9\times10^{9}\times2.5\times10^{-5}}{(0.18)^{2}}$

$= 6.94\times10^{5}\mathrm{V/m}$

The resultant field intensity at Z,

$E=\sqrt{E_A^{2}+E_B^{2}+2E_AE_B\cos2\theta}$

Where, 2θ is the angle, $\angle AZB$

From the figure, we obtain

$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$
$\theta = \cos^{-1}(0.5556) = 56.25^\circ$
$\therefore 2\theta = 112.5^\circ$
$\cos 2\theta = -0.38$
$E = \sqrt{\left(4.17\times10^5\right)^2 + \left(6.94\times10^5\right)^2 + 2\left(4.17\times10^5\right)\left(6.94\times10^5\right)(-0.38)}$
$= 6.6\times10^5 \mathrm{V/m}$
Therefore, the potential at a point 10 cm (perpendicular to the mid-point) is $2.0\times10^5 V$ and electric field is $6.6\times10^5 \mathrm{V/m}$.

15: A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q

a. **A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?**
b. **Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.**

Ans:

a. Charge placed at the centre of a shell is +q. Hence, a charge of magnitude –q will be induced to the inner surface of the shell and a charge of +q is induced on the outer surface of the shell.
   The total charge on the inner surface of the shell is –q.
   Surface charge density at the inner surface of the shell is given by the relation,
   $\sigma_1 = \frac{\text{Charge}}{\text{Surfacearea}} = \frac{-q}{4\pi r_1^2}$
   A charge of +q is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is Q + q. Surface charge density at the outer surface of the shell,
   $\sigma_2 = \frac{\text{Charge}}{\text{Surfacearea}} = \frac{Q+q}{4\pi r_2^2}$
b. Yes, the electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. The net charge on the inner surface enclosing the cavity is zero, since a charge placed in the cavity induces an equal and opposite charge on the inner

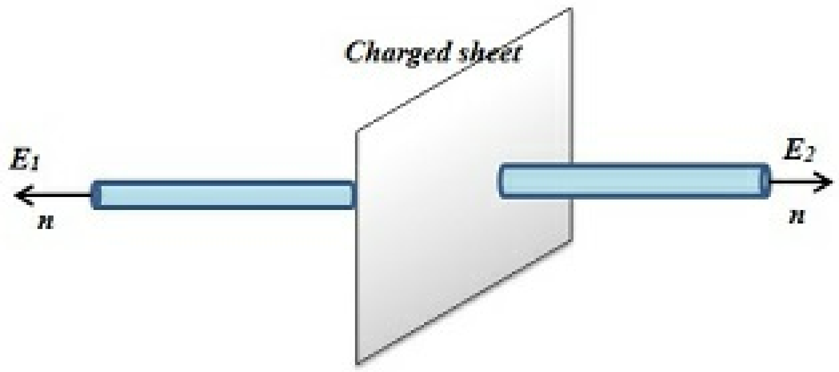
surface, irrespective of the size or shape of the surface. By Gauss theorem, the electric field inside a cavity is zero.

16:

a. **Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by** $\left(\vec{E}_1 - \vec{E}_2\right) \cdot \hat{n} = \frac{\sigma}{\varepsilon_0}$ **Where** $\hat{n}$ **is a unit vector normal to the surface at a point and** σ **is the surface charge density at that point. (The direction of** $\hat{n}$ **is from side 1 to side 2). Hence show that just outside a conductor, the electric field is** $\frac{\sigma}{\varepsilon_0}\hat{n}$
b. **Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]**

Ans:

a. Electric field on one side of a charged body is E_1 and electric field on the other side of the same body is E_2.



If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,
$\vec{E}_1 = \frac{-\sigma}{2\varepsilon_0}\hat{n}$
Where,
$\hat{n}$ = Unit vector normal to the surface at a point
σ = Surface charge density at that point.
Electric field due to the other surface of the charged body,
$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$

Electric field at any point due to the two surfaces,
$\left(\vec{E}_1-\vec{E}_2\right)=\frac{\sigma}{2\varepsilon_0}\hat{n}+\frac{\sigma}{2\varepsilon_0}\hat{n}=\frac{\sigma}{\varepsilon_0}\hat{n}$
$\left(\vec{E}_1-\vec{E}_2\right)\cdot\hat{n}=\frac{\sigma}{\varepsilon_0}$
Since E_1 and E_2 act in opposite directions, there is a discontinuity at the sheet of charge.
Since inside a closed conductor, $\vec{E}_1=0$,
Therefore, the electric field just outside the conductor is .$\vec{E}=\vec{E}_2=\frac{\sigma}{\varepsilon_0}\hat{n}$

b. Work done in an electric field over a closed path of length l is given by,
$W=\int\vec{E}\cdot d\vec{l}=\left(\vec{E}_1-\vec{E}_2\right)\cdot l$
$=E_1\cos\theta_1 l-E_2\cos\theta_2 l$
The tangential components of the vectors E_1 and E_2 are $E_1\cos\theta_1$ and $E_2\cos\theta_2$ respectively.
When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero.
$(E_1\cos\theta_1-E_2\cos\theta_2)\,l=0$

c. Therefore, $E_1\cos\theta_1=E_2\cos\theta_2$

Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

17: A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Ans: Charge density of the long charged cylinder of length L and radius r is λ.
Another cylinder of same length surrounds the pervious cylinder. The radius of this cylinder is R.
Let E be the electric field produced in the space between the two cylinders.
Electric flux through the Gaussian surface is given by Gauss's theorem as,
$\phi=E(2\pi d)L$
Where, d = Distance of a point from the common axis of the cylinders Let q be the total charge on the cylinder.
It can be written as

$\therefore \phi = E(2\pi dL) = \frac{q}{\epsilon_0}$

Where,

q = Charge on the inner sphere of the outer cylinder

$\in_0$ = Permittivity of free space

$E(2\pi dL) = \frac{\lambda L}{\epsilon_0}$

$E = \frac{\lambda}{2\pi\epsilon_0 d}$

$= \frac{\lambda}{2\pi\epsilon_0 d}$

Therefore, the electric field in the space between the two cylinders is $\frac{\lambda}{2\pi\varepsilon_0 d}$.

18: In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 $\overset{o}{A}$.

i. **Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.**
ii. **What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?**
iii. **What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 $\overset{o}{A}$ separation?**

Ans: The distance between electron-proton of a hydrogen atom, d = 0.53A

Charge on an electron, $q_1 = 1.6 \times 10^{-19} C$

Charge on a proton, $q_2 = +1.6 \times 10^{-19} C$

i. Potential at infinity is zero.
   $= 0 - \frac{q_1 q_2}{4\pi\epsilon_0 d}$
   Potential energy of the system, PE = Potential energy at infinity - Potential energy at distance d
   Where,
   $\in_0$ is the permittivity of free space
   $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{Nm}^2 C^{-2}$

$= 0 - \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{0.53 \times 10^{10}}$

$\therefore$ Potential energy $= -43.7 \times 10^{-19} J$

Since $1.6 \times 10^{-19} J = 1eV$,

$\therefore$ Potential energy $= \frac{-43.7 \times 10^{-19}}{1.6 \times 10^{-19}}$

= -27.2eV

Therefore, the potential energy of the system is -27.2eV.

ii. Kinetic energy is half of the magnitude of potential energy.

Kinetic energy $K = \frac{1}{2}(27.2) = 13.6eV$

Total energy = 13.6-27.2 = -13.6 eV

Therefore, the minimum work required to free the electron is 13.6 eV.

iii. When zero of potential energy is taken, d_1 = 1.06A

$\therefore$ Potential energy of the system = Potential energy at d1 - Potential energy at d

$= \frac{q_1 q_2}{4\pi\epsilon_0 d_1} - 27.2eV$

$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{1.06 \times 10^{-10}} - 27.2\text{eV}$

$= 21.73 \times 10^{-10} J - 27.2eV$

= 13.58 eV - 27.2 eV

= 13.6 eV

19: If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 $\overset{o}{A}$, and the electron is roughly 1$\overset{o}{A}$ from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Ans: The system of two protons and one electron is represented in the given figure.

Charge on proton 1, $q_1 = 1.6 \times 10^{-19} C$

Charge on proton 2, $q_2 = 1.6 \times 10^{-19} C$
Charge on electron, $q_3 = -1.6 \times 10^{-19}$C
Distance between protons 1 and 2, $d_1 = 1.5 \times 10^{-10}$m

Distance between proton 1 and electron, $d_2 = 1 \times 10^{-10}$m

Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10}$m

The potential energy at infinity is zero.

Potential energy for system of 3 charges is given by
$V = \frac{q_1 q_2}{4\pi\epsilon_0 d_1} + \frac{q_2 q_3}{4\pi\in_0 d_3} + \frac{q_3 q_1}{4\pi\epsilon_0 d_2}$
Substituting $\frac{1}{4\pi\in_0 d} = 9 \times 10^9 \mathrm{Nm}^2 C_2^{-2}$
$V = \frac{9\times10^9\times10^{-19}\times10^{-19}}{10^{-10}}\left[-(16)^2 + \frac{(1.6)^2}{1.5} + -(1.6)^2\right]$
$= -30.7 \times 10^{-19} J$
= -19.2eV
Therefore, the potential energy of the system is -19.2eV.

20: Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Ans: Let a be the radius of a sphere A, Q_A be the charge on the sphere, and C_A be the capacitance of the sphere. Let b be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let $E_A$be the electric field of sphere A and E_B be the electric field of sphere B. Therefore, their ratio,
$\frac{E_A}{E_B} = \frac{Q_A}{4\pi\in_0 \times a_2} \times \frac{b^2 4\pi\epsilon_0}{Q_S}$
$\frac{E_A}{E_B} = \frac{Q_A}{Q_s} \times \frac{b^2}{a^2}$.........(1)
However $\frac{Q_A}{Q_B} = \frac{C_A V}{C_s V}$
And $\frac{C_A}{C_B} = \frac{a}{b}$
$\therefore \frac{Q_A}{Q_B} = \frac{q}{b}$........(2)

Putting the value of (2) in (1), we obtain

$$\therefore \frac{E_A}{E_B} = \frac{a}{b}\frac{b^2}{a^2} = \frac{b}{a}$$

Therefore, the ratio of electric fields at the surface is $\frac{b}{a}$.

Since the electric field intensity due to a conductor is directly proportional to its charge density,

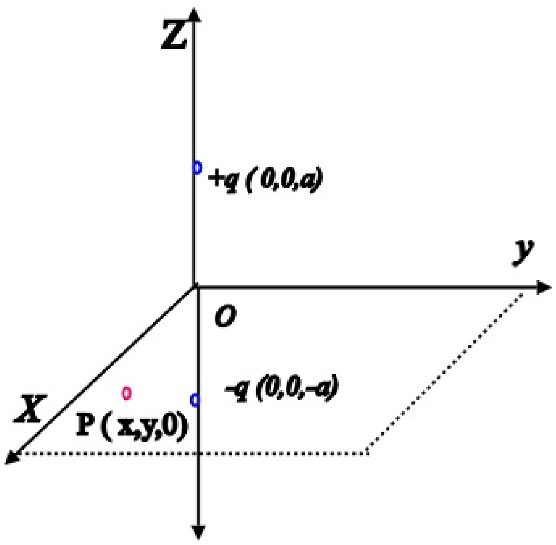
$$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{b}{a}$$

The charge density of a conductor is inversely proportional to its radius. Smaller the radius, greater is the charge density of the surface. Sharp points on a conductor have small radii and hence the charge density at such points is greater than the charge density on flatter parts of the surface.

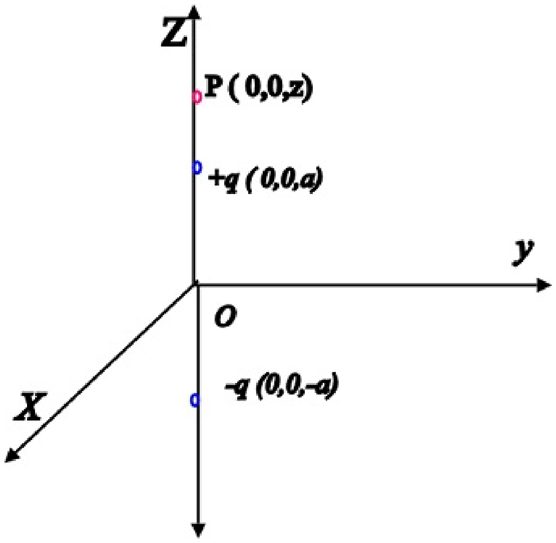
21: Two charges -*q* and +*q* are located at points (0,0,-a) and (0,0,a) respectively.

i. **What is the electrostatic potential at the points (0,0,z) and (x,y,0)?**
ii. **Obtain the dependence of potential on the distance r of a point from the origin when r/a >> 1.**
iii. **How much work is done in moving a small test charge from the point (5, 0, 0) to (-7,0,0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?**

Ans:

i. 

Charge - q is located at (0, 0, -a) and charge + q is located at (0, 0, a). Hence, they form a dipole. Point (0, 0, z) is on the axis of this dipole and point (x, y, 0) is normal to the axis of the dipole. Hence, electrostatic potential at point (x, y, 0) is **zero**.



The point *P* lies at a point where z>a

Electrostatic potential at point (0, 0, z) is given by,

$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z-a} + \frac{-q}{z-(-a)}\right)$

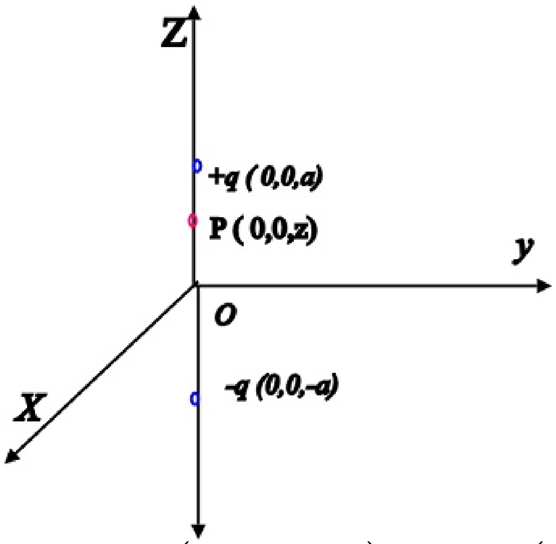
$= \frac{q}{4\pi\varepsilon_0}\left(\frac{2a}{z^2-a^2}\right) = \frac{p}{4\pi\varepsilon_0\left(z^2-a^2\right)}$

Where,

$\in_0$ = Permittivity of free space

p = Dipole moment of the system of two charges = 2qa

If z<a,



$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a-z} + \frac{-q}{a+z}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2z}{a^2-z^2}\right)$

If the point lies at a distance *z* closer to -q, the potential is same as $\frac{p}{4\pi\varepsilon_0(z^2-a^2)}$

ii. Distance r is much greater than half of the distance between the two charges.

Let *z* = *r* in the equation $V = \frac{p}{4\pi\varepsilon_0\left(z^2-a^2\right)}$

$V = \frac{p}{4\pi\varepsilon_0(r^2-a^2)}$

For $r >> a$,

$V = \frac{p}{4\pi\varepsilon_0 r^2}$

Hence, the potential (V) at a distance r is inversely proportional to square of the distance i.e., $V \propto \frac{1}{r^2}$

iii. Zero

A test charge is moved from point (5, 0, 0) to point (7, 0, 0) along the x-axis. Electrostatic potential (V_1) at point (5, 0, 0) is given by,

$V_1 = \frac{-q}{4\pi\epsilon_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}} + \frac{q}{4\pi\in_0}\frac{1}{(5-0)^2+a^2}$

$= \frac{-q}{4\pi\epsilon_0\sqrt{25^2+a^2}} + \frac{q}{4\pi\epsilon_0\sqrt{25+a^2}}$

= 0

Electrostatic potential, V_2, at point (7, 0, 0) is given by,

$V_2 = \frac{-q}{4\pi\epsilon_0}\frac{1}{\sqrt{(-7)^2+(-a)^2}} + \frac{q}{4\pi\in_0}\frac{1}{\sqrt{(-7)^2+(a)^2}}$

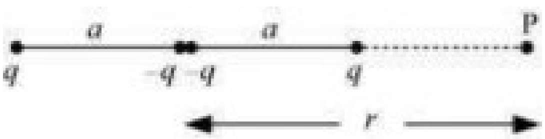
$= \frac{-q}{4\pi\epsilon_0\sqrt{49+a^2}} + \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{49+a^2}}$

= 0

Hence, no work is done in moving a small test charge from point (5, 0, 0) to point (7, 0, 0) along the x-axis.

The answer does not change if the path of the test is not along the x-axis because work done by the electrostatic field in moving a test charge between the two points is independent of the path connecting the two points.

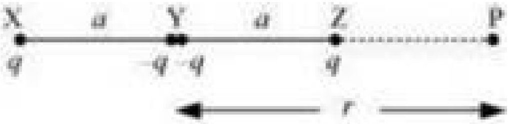
22: Figure shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



Ans: Four charges of same magnitude are placed at points X, Y, Y, and Z respectively, as shown in the following figure.

A point is located at P, which is r distance away from point Y. The system of charges forms an electric quadrupole.

It can be considered that the system of the electric quadrupole has three charges.



Charge +q placed at point X

Charge -2q placed at point Y

Charge +q placed at point Z

XY = YZ = a

YP = r

PX = r + a

PZ = r - a

Electrostatic potential caused by the system of three charges at point P is given by,

$V = \frac{1}{4\pi\in_0}\left[\frac{q}{XP} - \frac{2q}{YP} + \frac{q}{ZP}\right]$

$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{r(r-a)-2(r+a)(r-a)+r(r+a)}{r(r+a)(r-a)}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2-ra-2r^2+2a^2+r^2+ra}{r(r^2-a^2)}\right]$

$= \frac{q}{4\pi\epsilon_0}\left[\frac{2a^2}{r(r^2-a^2)}\right]$

$= \frac{2qa^2}{4\pi\in_0 r^3\left(1-\frac{a^2}{r^2}\right)}$

Since $\frac{r}{a} >> 1,$

$\therefore \frac{r}{a} >> 1$

$\frac{r^2}{a^2}$ is taken as negligible.

$\therefore V = \frac{2qa^2}{4\pi\epsilon_0 r^3}$

It can be inferred that potential, $V \propto \frac{1}{r^3}$

However, it is known that for a dipole, $V \propto \frac{1}{r^2}$

And, for a monopole, $V \propto \frac{1}{r}$.

23: An electrical technician requires a capacitance of 2µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V.

Suggest a possible arrangement that requires the minimum number of capacitors.

Ans: Total required capacitance, C =2μF Potential difference, V = 1 kV = 1000 V Capacitance of each capacitor,C_1 = 1 μF

Each capacitor can withstand a potential difference, V1 = 400 V

Let n capacitors be connected in series and these series circuits are connected in parallel forming m rows . The potential difference across each row must be 1000 V and potential difference across each capacitor must be 400 V. Hence, number of capacitors in each row is given as

$n = \frac{1000}{400} = 2.5$

Since the number of capacitors cannot be in fractions, therefore, there are 3 capacitors in each row.

Capacitance of each row $\frac{1}{C_n} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$

$C_n = \frac{1}{3}\mu\mathrm{F}$

If m such capacitors are connected in parallel, the equivalent capacitance of the combination is the needed value of capacitance of 2μF. Therefore, $C = \frac{1}{3} \times m = 2\mu\mathrm{F}$

$m = 6$

Let there are 6 rows, each having 3 capacitors, which are connected in parallel. Hence, the total number of capacitors required is 6 $\times$ 3 = 18

24: What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realize from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Ans: Capacitance of a parallel capacitor, V = 2 F

Distance between the two plates, d = 0.5 cm = $0.5 \times 10^{-2}\mathrm{m}$ Capacitance of a parallel plate capacitor is given by the relation,

Where,

$\in_0$ = Permittivity of free space $= 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

$\therefore A = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}}$

= $1130 km^2$

Hence, the area of the plates is too large. To avoid this situation, the capacitance is taken in the range of micro Farads.

25: Obtain the equivalent capacitance of the network in Fig. 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.

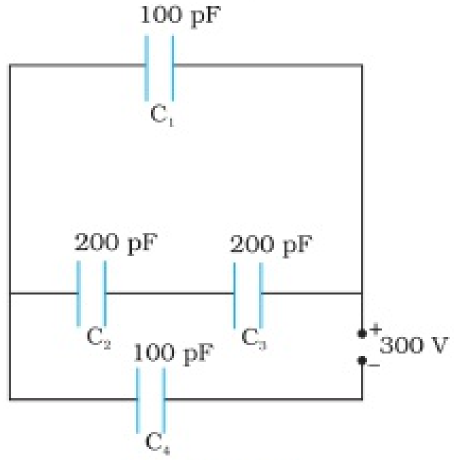


FIGURE 2.35

Ans: Capacitance of capacitor C_1 is 100 pF.

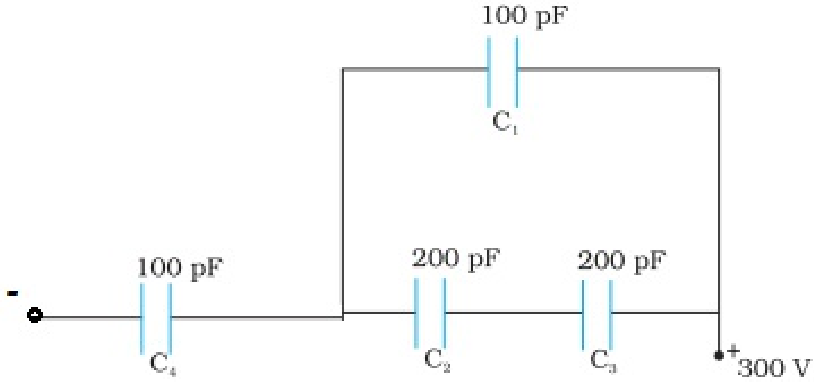
Capacitance of capacitor C_2 is 200 pF.

Capacitance of capacitor C_3 is 200 pF.

Capacitance of capacitor C_4 is 100 pF.

Supply potential, V = 300 V

The circuit is redrawn as below



Capacitors C_2 and C_3 are connected in series. Let their equivalent capacitance be C'

$\therefore \frac{1}{C^1} = \frac{1}{200} + \frac{1}{200} = \frac{2}{200}$

C^1 = 100pF

Capacitors $C_1$and C' are in parallel. Let their equivalent capacitance be C^n

$\therefore C^n = C^1 + C_1$

= 100 + 100 = 200pF

C^n and $C_4$are connected in series. Let their equivalent capacitance be C.

$\therefore \frac{1}{C} = \frac{1}{C^n} + \frac{1}{C_4}$
$= \frac{1}{200} + \frac{1}{100} = \frac{2+1}{200}$
$C = \frac{200}{3} pF$

Hence, the equivalent capacitance of the circuit is $\frac{200}{3}\mathrm{pF}$

The total charge stored in the network is

$q = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8}\mathrm{C}$

The charge on C_4 is $q = q_4 = 2 \times 10^{-8}\mathrm{C}$

The potential difference across C_4 is

$V_4 = \frac{q_4}{C_4} = \frac{2\times10^{-8}}{100\times10^{-12}} = 200\mathrm{V}$

The potential difference across the parallel combination is V_1= 300 - 200 =100 V

The charge on C_1 is $q_1 = C_1 V_1 = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}\mathrm{C}$

Since C_2= C_3, the potential difference across the capacitors are equal.

$V_2 = V_3 = \frac{V_1}{2} = \frac{100}{2} = 50\mathrm{V}$

The charges stored are also equal ,

$q_2 = q_3 = 200 \times 10^{-12} \times 50 = 1 \times 10^{-8}\mathrm{C}.$

26: The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

a. **How much electrostatic energy is stored by the capacitor?**
b. **View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.**

Ans: Area of the plates of a parallel plate capacitor, A = 90 $\mathrm{cm}^2 = 90 \times 10^{-4} m^2$

Distance between the plates, d = 2.5 mm $= 2.5 \times 10^{-3}\mathrm{m}$

Potential difference across the plates, V = 400 V

a. Capacitance of the capacitor is given by the relation,
$C = \frac{\epsilon_0 A}{d}$
Electrostatic energy stored in the capacitor is given by the relation,$U_1 = \frac{1}{2}CV^2$
$= \frac{1}{2}\frac{\epsilon_0 A}{d}V^2$
Where,
$\in_0$ = Permittivity of free space = $8.85 \times 10^{-12}\mathrm{C}^2 N^{-1} m^{-2}$
$U_1 = \frac{\left(8.85\times10^{-12}\right)\left(90\times10^{-4}\right)(400)^2}{\left(2\times2.5\times10^{-3}\right)} = 2.55 \times 10^{-6}\mathrm{J}$
Hence, the electrostatic energy stored by the capacitor is $2.55 \times 10^{-6}J$
b. Volume of the given capacitor,
$V^1 = A \times d$
$= 90 \times 10^{-4} \times 25 \times 10^{-3}$
$= 2.25 \times 10^{-4} m^3$
Energy stored in the capacitor per unit volume is given by
$u = \frac{U_1}{V'}$
$= \frac{2.55\times10^{-6}}{2.25\times10^{-4}} = 0.113\mathrm{Jm}^{-3}$
Where,
$u = \frac{U_1}{V'} = \frac{U_1}{Ad}$
$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 A}{d}\frac{V^2}{Ad}$
$= \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$
Since the electric field intensity $E = \frac{V}{d}$
$u = \frac{1}{2}\varepsilon_0 E^2$.

27: A 4μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Ans: Capacitance of a charged capacitor, $C_1 = 4\mu F = 4 \times 10^{-6}F$ Supply voltage, V_1 = 200 V

Electrostatic energy stored in C_1 is given by,

$U_1 = \frac{1}{2} C_1 {V_1}^2$
$= \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2$
$= 8 \times 10^{-2} J$
$C_2 = 2uF = 2 \times 10^{-6}\mathrm{F}$

Capacitance of an uncharged capacitor,

When C_2 is connected to the circuit, the potential acquired by it is V_2.

$: V_2 (C_1 + C_2) = C_1 V_1$
$V_2 \times (4+2) \times 10^{-6} = 4 \times 10^{-6} \times 200$
$V_2 = \frac{400}{3} V$

According to the conservation of charge, initial charge on capacitor C_1 is equal to the final charge on capacitors, C_1 and C_2.

Electrostatic energy for the combination of two capacitors is given by,

$U_2 = \frac{1}{2} (C_1 + C_2) {V_2}^2$
$= \frac{1}{2}(2+4) \times 10^{-5} \times \left(\frac{400}{3}\right)^2$
$= 5.33 \times 10^{-2} J$

Hence, amount of electrostatic energy lost by capacitor

$\Delta U = U_1 - U_2 = (8 - 5.33) \times 10^{-2} = 2.67 \times 10^{-2}\mathrm{J}.$

28: Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (1/2) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.

Ans: Let F be the force applied to separate the plates of a parallel plate capacitor by a distance of Δx. Hence, work done by the force to do so $F(\Delta x)$

As a result, the potential energy of the capacitor increases by an amount $uA\Delta x$.

Where,

u = Energy density

A = Area of each plate

d = Distance between the plates

V = Potential difference across the plates

The work done will be equal to the increase in the potential energy i.e.,

$F_{\Delta}x = uA\Delta x$

$F = uA = \left(\frac{1}{2}\epsilon_0 E^2\right)A$

Electric Field intensity is given by,

$E = \frac{V}{d}$

$\therefore F = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)EA = \frac{1}{2}\left(\epsilon_0 A\frac{V}{d}\right)E$

However, capacitance, $C = \frac{\in_0 A}{d}$

$\therefore F = \frac{1}{2}(CV)E$

Charge on the capacitor is given by,

Q = CV

$\therefore F = \frac{1}{2}QE$

The physical origin of the factor, $\frac{1}{2}$, in the force formula lies in the fact that just outside the conductor, field is E and inside it is zero. Hence, it is the average value, $\frac{E}{2}$, of the field that contributes to the force.

29: A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports . Show that the capacitance of a spherical capacitor is given by $C = \frac{4\pi\in_0 r_1 r_2}{r_1 - r_2}$ where r_1 and r_2 are the radii of outer and inner spheres, respectively.

Ans: Radius of the outer shell = r_1

Radius of the inner shell = r_2

The inner surface of the outer shell has charge +Q.

$V = \frac{Q}{4\pi\in_0 r_2} - \frac{Q}{4\pi\in_0 r_1}$

The outer surface of the inner shell has induced charge -Q. Potential difference between the two shells is given by,

Where, $\in_0$ = Permittivity of free space

$V = \frac{Q}{4\pi\in_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{Q(r_1 - r_2)}{4\pi\epsilon_0 r_1 r_2}$

Capacitance of the given system is given by,

$C = \frac{Charge(Q)}{Voltage(V)}.$

$= \frac{4\pi\in_0 r_1 r_2}{r_1 - r_2}$

Hence, proved.

30: A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

i. **Determine the capacitance of the capacitor.**
ii. **What is the potential of the inner sphere?**
iii. **Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.**

Ans: Radius of the inner sphere, r_2 = 12 cm = 0.12 m Radius of the outer sphere, r_1 = 13 cm = 0.13 m Charge on the inner sphere, $q = 2.5\mu C = 2.5 \times 10^{-6}$
Dielectric constant of a liquid, $\epsilon_r = 32$
(a) Capacitance of the capacitor is given by the relatioon,
$C = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$
Where,
$\in_0$ = Permittivity of free space = $8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$
$V = \frac{1}{4\pi\epsilon_0} 9 \times 10^9 \mathrm{Nm}^2\mathrm{C}^{-2}$
$\therefore C = \frac{32 \times 0.12 \times 0.13}{9 \times 10^9 \times (0.13 - .12)}$
$\approx 5.5 \times 10^{-9} F$
Hence, the capacitance of the capacitor is approximately .

i. Potential of the inner sphere is given by,
$V = \frac{q}{C}$
$= \frac{2.5 \times 10^{-6}}{5.5 \times 10^{-9}} = 4.5 \times 10^2 V$
Hence, the potential of the inner sphere is $4.5 \times 10^2 V$.
ii. Radius of an isolated sphere, $r = 12 \times 10^{-2} m$
iii. Capacitance of the sphere is given by the relation, $C^1 - 4\pi\epsilon_0 r$
$= 4\pi \times 8.85 \times 10^{-12} \times 12 \times 10^{-12}$
$= 1.33 \times 10^{-11} F$
The capacitance of the isolated sphere is less in comparison to the concentric spheres. This is because the outer sphere of the concentric spheres is earthed. Hence, the potential

difference is less and the capacitance is more than the isolated sphere.

31: Answer carefully:

i. **Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$, where r is the distance between their centres?**
ii. **If Columb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?**
iii. **A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?**
iv. **What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?**
v. **We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?**
vi. **What meaning would you give to the capacitance of a single conductor?**
vii. **Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).**

Ans:

i. Coulomb's law is valid for force between two point charges. The force between two conducting spheres is not exactly given by the expression $\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$, because as the spheres are brought closer, the charge distribution becomes non-uniform and they no longer behave as point charges.
ii. Gauss law involves integration over a surface for the determination of flux.
$\phi = \int d\phi = \oint \vec{E} \cdot d\vec{S}$
If Coulomb's law involved $1/r^3$ dependence, instead of$1/r^2$, Gauss law will not hold true since the flux would then depend on *r* too.
iii. Not necessarily; If a small test charge is released at rest at a point in an electrostatic field configuration, then it will travel along the field lines passing through the point, only if the field lines are straight. This is because the field lines give the direction of acceleration

and not of velocity.

iv. Whenever the electron completes an orbit, either circular or elliptical, the work done by the field of a nucleus is zero since the electrostatic field is a conservative field.

v. No Electric field is discontinuous across the surface of a charged conductor. However, an electric potential is continuous and has a constant value.

vi. The capacitance of a single conductor is considered as a parallel plate capacitor with one of its two plates at infinity.

vii. Water is assymetrical molecule when compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica.

32: A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Ans: Length of a co-axial cylinder, l = 15 cm = 0.15 m

Radius of outer cylinder, r_1 = 1.5 cm = 0.015 m

Radius of inner cylinder, r_2 = 1.4 cm = 0.014 m

Charge on the inner cylinder, q = 3.5 $\mu C = 3.5 \times 10^{-6}\mathrm{C}$

Capacitance of a co-axil cylinder of radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi\epsilon_0 1}{\log_2 \frac{r_1}{r_2}}$$

Where,

$\in_0$ = Permittivity of free space = $8.85 \times 10^{-12} N^{-1} m^{-2} C^2$

$$\therefore C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \log_{10}\left(\frac{0.15}{0.14}\right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299} = 1.2 \times 10^{-10} F$$

Potential difference of the inner cylinder is given by,

$$V = \frac{q}{C}$$

$$= \frac{3.5 \times 10 - 6}{1.2 \times 10^{-10}} = 2.92 \times 10^4 V.$$

33: A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7Vm^{-1}.

(Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Ans: Potential rating of a parallel plate capacitor, V = 1 kV = 1000 V

Dielectric constant of a material, $\in_r$ =3 Dielectric strength = 10^7 V/m

For safety, the field intensity never exceeds 10% of the dielectric strength. Hence, electric field intensity, E = 10% of 10^7 = 10^6V/m

Capacitance of the parallel plate capacitor, C = 50 pF $= 50 \times 10^{-12} F$

Distance between the plates is given by,

$d = \frac{V}{E}$

$= \frac{1000}{10^6} = 10^{-3} m$

Capacitance is given by the relation,

$C = \frac{\epsilon_0 \epsilon_1 A}{d}$

Where, A = Area of each plate

$\in_0$ = Permittivity of free space $= 8.85 \times 10^{-12} N^{-1} C^2 m^{-2}$

$\therefore A = \frac{CD}{\in_0 \in_r}$

$= \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3} \approx 19 \mathrm{cm}^2$

Hence, the area of each plate is about 19 cm^2.

34: Describe schematically the equipotential surfaces corresponding to

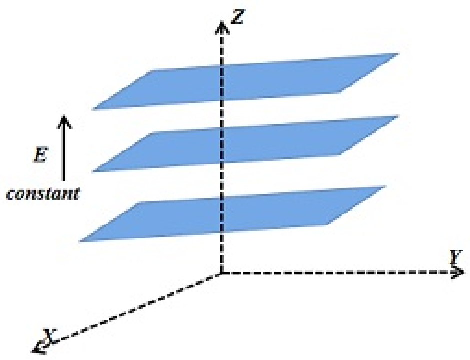
1. a constant electric field in the z-direction,

2. a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

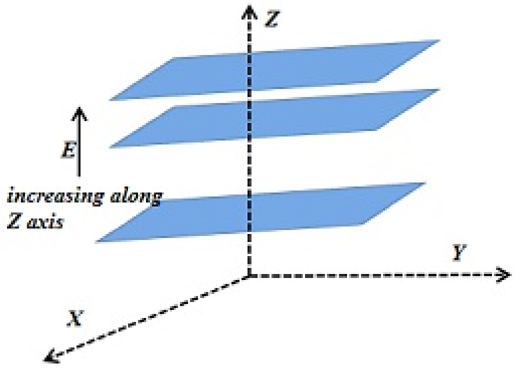
3. a single positive charge at the origin, and

4. a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Ans: 1. Equidistant planes parallel to the x-y plane are the equipotential surfaces.



2. Planes parallel to the x-y plane are the equipotential surfaces with the exception that when the planes get closer, the field increases.



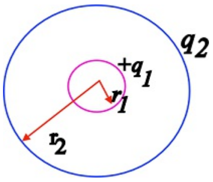
3. Concentric spheres centered at the origin are equipotential surfaces.
4. A periodically varying shape near the given grid is the equipotential surface. This shape gradually reaches the shape of planes parallel to the grid at a larger distance.

35: In a Van de Graaff type generator a spherical metal shell is to be a $15 \times 10^{6}\mathrm{V}$ electrode. The dielectric strength of the gas surrounding the electrode is $5 \times 10^{7}\mathrm{V}m^{-1}$. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

Ans: Potential difference, V = $15 \times 10^{6}\mathrm{V}$
Dielectric strength of the surrounding gas = $5 \times 10^{7} V/\mathrm{m}$
Electric field intensity, E = Dielectric strength = $5 \times 10^{7} V/\mathrm{m}$
Minimum radius of the spherical shell required for the purpose is given by,
$r = \frac{V}{E} = \frac{15\times10^{6}}{5\times10^{7}} = 0.3m = 30\mathrm{cm}$
Hence, the minimum radius of the spherical shell required is 30 cm.

36: A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Ans: Consider a sphere of radius r_1 carrying a charge q_1 placed inside a spherical shell of radius r_2 carrying a charge q_2.



The total potential of the outer sphere V_O = potential due to the charge q_2 on its surface + potential due to charge q_1 on the inner sphere.

$$V_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_2} + \frac{q_1}{r_2}\right)$$

The potential on the inner sphere is V_i = potential due to the charge q_1 on its surface and the potential due to charge q_2 on the outer sphere. The potential due to the charge on the outer sphere is constant through out the volume of the hollow spherical shell. Therefore,

$$V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

The potential difference between the inner and the outer conductors is given by

$$V_i - V_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_2} + \frac{q_1}{r_2}\right) = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

since $r_1 < r_2$, V_i - V_O will be positive if q_1 is positive. The charge therefore will always flow from the inner sphere to the outer sphere if both are connected , irrespective of the value of the charge on the outer sphere.

37: Answer the following:

i. **The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100V/m. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)**
ii. **A man fixes outside his house one evening a two metre high insulating slab**

carrying on its top a large aluminium sheet of area $1m^2$. Will he get an electric shock if he touches the metal sheet next morning?

iii. **The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?**

iv. **What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning? (Hint: The earth has an electric field of about $100Vm^{-1}$ at its surface in the downward direction, corresponding to a surface charge density = $-10^{-9}C\ m^{-2}$. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about +1800C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)**

Ans:

i. We do not get an electric shock as we step out of our house because the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential. This is because our body is a good conductor.
ii. Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminum sheet. As a result, its voltage rises gradually. The raise in the voltage depends on the capacitance of the capacitor formed by the aluminium slab and the ground.
iii. The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.
iv. During lightning and thunderstorm, light energy, heat energy, and sound energy are dissipated in the atmosphere.

CBSE Class 12 Physics

NCERT Solutions

Chapter - 3

Current Electricity

1: The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is Ω 0.4, what is the maximum current that can be drawn from the battery?

Ans: Emf of the battery, E = 12 V

Internal resistance of the battery, r = 0.4 Ω

Maximum current that can be drawn from the battery, I= E/r= 12/0.4=30 A

So, the maximum current drawn from the given battery is 30 A.

2: A battery of emf 10 V and internal resistance 3Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

Ans: Emf of the battery, E=10 V

Internal resistance of the battery ,r =3Ω

Current in the circuit, I=0.5 A

Resistance of the resistor = R

As by Ohm's law

$$I=\frac{E}{R+r}$$
$$R+r=\frac{E}{I}$$
$$=\frac{10}{0.5}=20\Omega$$
$$\therefore R=20-3=17\Omega$$

Terminal voltage of the resistor=V

Using Ohm's law,

V = IR

$$=0.5\times 17$$

= 8.5 V

Therefore, the resistance of the resistor is 17Ω and the terminal voltage is 8.5 V.

3:

a. **Three resistors** $1\Omega, 2\Omega$ and 3Ω **are combined in series. What is the total resistance of the combination?**
b. **If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.**

Ans:

a. Three resistors of resistances $1\Omega, 2\Omega$ and 3Ω are combined in series. In series combination of resistances ,total resistance of the combination is given by the formula as
   $R = R_1 + R_2 + R_3$
   So total resistance=1 + 2 + 3 = 6Ω
b. Current flowing through the circuit = I
   Emf of the battery, E=12 V
   Total resistance of the circuit by Ohm's law is given as,
   $I = \frac{E}{R} = \frac{12}{6} = 2\text{A}$
   Let potential drop across 1Ω resistor = V_1
   by Ohm's law
   $V_1 = 2 \times 1 = 2V$ .... (i)
   Let potential drop across 2Ω resistor = V_2
   by ohm's law
   $V_2 = 2 \times 2 = 4V$ ...... (ii)
   Let potential drop across 3Ω resistor = V_3
   By ohm's law
   $V_3 = 2 \times 3 = 6V$ ...... (iii)
c. Therefore, the potential drop across the $1\Omega, 2\Omega \text{and} 3\Omega$ resistors are 2V, 4V, and 6V respectively.

4.

a. **Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total**

resistance of the combination?

b. **If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.**

Ans:

a. There are three resistors of resistances,

R_1 = 2, R_2 = 4, and R_3 = 5

They are connected in parallel. Hence, total resistance(R) of the combination of parallel resistances is given by,

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$
$$=\frac{1}{2}+\frac{1}{4}+\frac{1}{5}=\frac{10+5+4}{20}=\frac{19}{20}$$
$$\therefore R=\frac{20}{19}\Omega$$

Therefore, total resistance of the combination is $\frac{20}{19}\Omega$.

b. Emf of the battery, V = 20V

Let currents $I_1,\ I_2\ and\ I_3$ flow through the resistors R_1, R_2 and R_3 and is given

$$I_1=\frac{V}{R_1}=\frac{20}{2}=10$$
$$I_2=\frac{V}{R_2}=\frac{20}{4}=5A$$
$$I_3=\frac{V}{R_3}=\frac{20}{5}=4\text{A}$$

Total current, I = I_1 + I_2 + I_3 = 10 + 5+ 4 = 19 A

Therefore, the current through each resistor is 10A, 5A, and 4A, respectively and the total current is 19A.

5: At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is $1.70\times10^{-4}\ {}^{0}\text{C}^{-1}$**?**

Ans: Room temperature, T=27^{o}C

Resistance of the heating element at temperature T_1 *is* $R_1=117\Omega$

Temperature co-efficient of the material of the filament,

$\alpha = 1.70 \times 10^{-4}\ {}^{0}\mathrm{C}^{-1}$

α is given by the relation,

$\alpha = \frac{R_1 - R}{R(T_1 - T)}$

$T_1 - T = \frac{R_1 - R}{R\alpha}$

$T_1 - 27 = \frac{117 - 100}{100(1.7 \times 10^{-4})}$

T_1 - 27 = 1000

Therefore, at $1027^{\circ}C$ the resistance of the element is 117Ω.

6: A negligibly small current is passed through a wire of length 15 m and uniform cross section $= 6.0 \times 10^{-7}\mathrm{m}^2$ and its resistance is measured to be 5.0Ω. What is the resistivity of the material at the temperature of the experiment?

Ans: Resistivity of material is given by $\rho = \frac{Ra}{l}$

Length of the wire, l =15 m

Area of cross-section of the wire,$a = 6.0 \times 10^{-7}\mathrm{m}^2$

Resistance of the material of the wire, $R = 5.0\Omega$

Resistivity of the material of the wire= ρ

Resistivity of material of wire is given as:

$\rho = \frac{Ra}{l} = \frac{5 \times 6 \times 10^{-7}}{15}$

$= 2 \times 10^{-7}\Omega m$

Therefore, the resistivity of the material is $2 \times 10^{-7}\Omega\mathrm{m}$.

7: A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver.

Ans: Temperature, T_1 = 27.5°C

Resistance of the silver wire at temperature T_1 *is* $R_1 = 2.1\Omega$

Temperature, T_2 = 1000°C

Resistance of the silver wire at temperature T_2 *is* $R_2 = 2.7\Omega$

Let temperature coefficient of resistance of silver =α

It is related with temperature and resistance as

$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)} = \frac{2.7 - 2.1}{2.1(100 - 27.5)}$

= $0.0039^{o}C^{-1}$

Therefore, the temperature coefficient of silver is $0.0039^{o}C^{-1}$.

8: A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $= 1.7 \times 10^{-4} C^{-1}$

Ans: Supply voltage, V=230V

Initial current drawn=I_1 = 3.2A

Initial resistance= R_1 which is given by the relation,

$R_1 = \frac{V}{I} = \frac{230}{3.2} = 71.87\Omega$

Steady state value of the current, I_2 = 2.8A

Resistance at the steady state = R_2 which is given as

$R_2 = \frac{230}{2.8} = 82.14\Omega$

Temperature co-efficient of resistance of nichrome, $\alpha = 1.70 \times 10^{-4}\mathrm{C}^{-1}$

Initial temperature of nichrome, T_1 = 27.0°C

Steady state temperature reached by nichrome= T_2

Since,

$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$

Therefore,

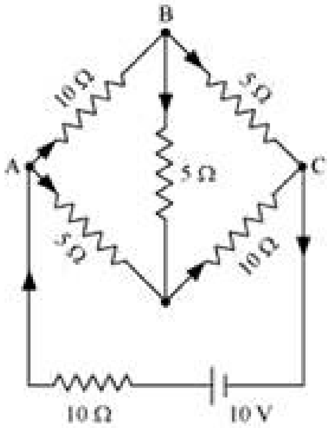
$T_2 - 27^{\circ}C = \frac{82.41 - 71.87}{71.87 \times 1.7 \times 10^{-4}} = 840.5$

T_2 = 840.5 + 27 = 867.5° C

Therefore, the steady temperature of the heating element is $867.5^{o}C$.

9: Determine the current in each branch of the network shown in fig 3.30:

Ans: Current flowing through various branches of the circuit is represented in the given figure.



I_1 = Current flowing through the outer circuit

I_2 = Current flowing through branch AB

I_3 = Current flowing through branch AD

$I_2 - I_4$ = Current flowing through branch BC

$I_3 + I_4$ = Current flowing through branch CD

I_4 = Current flowing through branch BD

Applying Kirchhoff's loop rule to the closed circuit ABDA,

$10I_2 + 5I_4 - 5I_3 = 0$

$2I_2 + I_4 - I_3 = 0$

$I_3 = 2I_2 + I_4$ ..... (1)

Applying Kirchhoff's 2nd law to the closed circuit BCDB,

$5(I_2 + I_4) - 10(I_3 + I_4) - 5I_4 = 0$

$5I_2 + 5I_4 - 10I_3 - 10I_4 - 5I_4 = 0$

$5I_2 - 10I_3 - 20I_4 = 0$

$I_2 = 2I_3 + 4I_4$..........(2)

Applying Kirchhoff's 2nd law to the closed circuit ABCFEA,

$-10 + 10(I_1) + 10(I_2) + 5(I_2 - I_4) = 0$

$10 = 15I_2 + 10I_1 - 5I_4$

$3I_2 + 2I_1 - I_4 = 2$..........(3)

From equations (1) and (2), we obtain

$I_3 = 2(2I_3 + 4I_4) + I_4$

$I_3 = 4I_3 + 8I_4 + I_4$

$-3I_3 = 9I_4$

$-3I_4 = +I_3$ ...... (4)

Putting equation (4) in equation (1), we obtain

$I_3 = 2I_2 + I_4$

$-4I_4 = 2I_2$

$I_2 = -2I_4$ ...... (5)

It is evident from the given figure that,

$I_1 = I_3 + I_2$ .... (6)

Putting equation (6) in equation (1), we obtain

$3I_2 + 2(I_3 + I_2) - I_4 = 2$

$5I_2 + 2I_3 - I_4 = 2$...... (7)

Putting equations (4) and (5) in equation (7), we obtain

$$5\left(-2I_4\right) + 2\left(-3I_4\right) - I_4 = 2$$
$$10I_4 - 6I_4 - I_4 = 2$$

$17I_4 = -2$

Equation (4) reduces to

$$I_3 = -3\left(I_4\right)$$
$$= -3\left(\frac{-2}{17}\right) = \frac{6}{17}A$$
$$I_2 = -2\left(I_4\right)$$
$$= -2\left(\frac{-2}{17}\right) = \frac{4}{17}A$$
$$I_2 - I_4 = \frac{4}{17} - \left(\frac{-2}{17}\right) = \frac{6}{17}\mathrm{A}$$
$$I_3 + I_4 = \frac{6}{17} + \left(\frac{-2}{17}\right) = \frac{4}{17}\mathrm{A}$$
$$I_1 = I_3 + I_2$$
$$= \frac{6}{17} + \frac{4}{17} = \frac{10}{17}\mathrm{A}$$

Therefore, current in branch AB $= \frac{4}{17}\mathrm{A}$

current in branch BC $= \frac{6}{17}\mathrm{A}$

current in branch CD $= \frac{-4}{17}\mathrm{A}$

current in branch AD$= \frac{6}{17}\mathrm{A}$

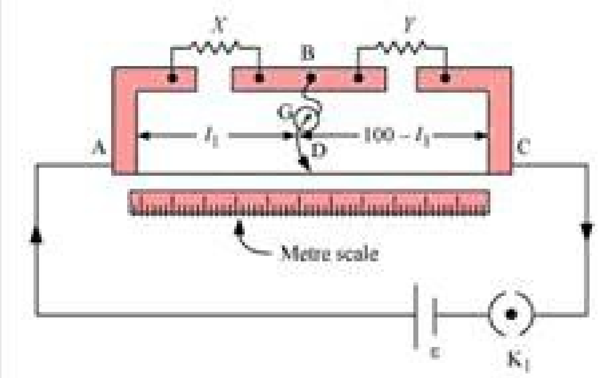
current in branch BD$= \frac{-2}{17}\mathrm{A}$

Therefore total current = $\frac{4}{17} + \frac{6}{17} + \frac{-4}{17} + \frac{6}{17} + \frac{-2}{17} = \frac{10}{17}\mathrm{A}$.

10:

a. **In a metre bridge [Fig. 3.27], the balance point is found to be at 39.5 cm from the end when the resistor Y is of 12.5 Ω.**
b. **Determine the balance point of the given bridge if X and Y are interchanged.**
c. **What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?**

Ans: A metre bridge with resistors X and Y is represented in the given figure.



a. Balance point from end A is l_1 =39.5 cm

Resistance of the resistor Y $= 12.5\Omega$

Condition for the balance is given as,

$$\frac{X}{Y} = \frac{l_1}{100 - l_1}$$
$$\frac{X}{Y} = \frac{39.5}{100-39.5} \times 12.5 \ = 8.2\Omega$$

Therefore, the resistance of resistor X is 8.2Ω.

b. If X and Y are interchanged, then, l_1 and (100 -l_1) get interchanged.

The balance point of the bridge will be at a distance (100 -l_1) from A.

100 -l_1 = 100 − 39.5 = 60.5 cm

Therefore, the balance point is 60.5 cm from A.

c. When the galvanometer and cell are interchanged at the balance point of the bridge, the galvanometer will show no deflection so no current would flow through the galvanometer.

11: A storage battery of emf 8.0 V and internal resistance 0.5Ω is being charged

by a 120 V dc supply using a series resistor of 15.5Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

Ans: Emf of the storage battery, E

Internal resistance of the battery, r =0.5Ω

DC supply voltage, V = 120 V

Resistance of the resistor, R = 15.5Ω

Effective voltage in the circuit =V_1

R is connected to the storage battery in series. Hence, it can be written as

V_1 = V - E

V_1 = 120 - 8 = 112V

Current flowing in the circuit can be calculated as: $I = \frac{V_1}{R+r} = \frac{112}{15.5+0.5} = 7\ A$

Voltage across resistor R is given by the product , IR= 7 $\times$ 15.5 = 108.5 V

DC supply voltage = Terminal voltage of battery + Voltage drop across R

Terminal voltage of battery = 120 - 108.5 = 11.5 V.

A series resistor in a charging circuit limits the current drawn from the external source and saves the circuit from extremely high current which is very dangerous.

12: In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

Ans: Emf of the cell, E_1 =1.25 V

Balance point of the potentiometer, I_1 = 35 cm

The cell is replaced by another cell of emf E_2

New balance point of the potentiometer, I_2 =63 cm

By the condition of balance of potentiometer

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$E_2 = E_1 \times \frac{l_2}{l_1}$$

$$= 1.25 \times \frac{63}{35} = 2.25V$$

Therefore, emf of the second cell is 2.25 V.

13: The number density of free electrons in a copper conductor estimated in Example is $8.5 \times 10^{28} m^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross section of the wire is $2 \times 10^{28} m^{-3}$ and it is carrying a current of 3.0 A.

Ans: Number density of free electrons in a copper conductor, $n = 8.5 \times 10^{28} m^{-3}$

Length of the copper wire, l = 3.0 m

Area of cross-section of the wire, $A = 2.0 \times 10^{-6} m^2$

Current carried by the wire, I=A,

as $I = neAv_d$

Where,

e = Electric charge= $1.6 \times 10^{-19} C$

$v_d = \frac{l}{t}$

$I = n\text{A}e\frac{l}{t}$

$t = \frac{nAel}{I}$

= $\frac{3 \times 8.5 \times 10^{28} \times 2 \times 10^{-6} \times 1.6 \times 10^{-19}}{3}$

$= 2.7 \times 10^4 s$

Therefore, the time taken by an electron to drift form one end of wire to the other is $2.7 \times 10^4 s$

14: The earth's surface has a negative surface charge density of 10^{-9}cm^{-2}. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different part of the globe [Radius of earth = $6.37 \times 10^6 \text{m}$.

Ans: Surface charge density of the earth, $\delta = 10^{-9} Cm^{-2}$

Current over the entire globe, I = 1800A
Radius of the earth, $r = 6.37 \times 10^6 m$
Surface area of the earth,
$A = 4\pi r^2$
$= 4 \times 3.14 \times \left(6.37 \times 10^6\right)^2$
$= 5.09 \times 10^{14} \mathrm{m}^2$
Charge on the earth surface = Surface charge density of the earth x Surface Area of the earth
$= 10^{-9} \times 5.09 \times 10^{14} = 5.09 \times 10^5 C$
Time taken to neutralize the earth's surface = t
$I = \frac{q}{t}$
$t = \frac{q}{I}$
$= \frac{5.09 \times 10^5}{1800} = 282.77 s$
Therefore, it takes 282.77 s to neutralize the earth's surface.

15:

a. **Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015Ω are joined in series to provide a supply to a resistance of 8.5Ω. What are the current drawn from the supply and its terminal voltage?**
b. **A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?**

Ans:

a. Number of secondary cells, n
   Emf of each secondary cell, E= 2.0V
   Internal resistance of each cell, r= 0.015Ω
   series resistor is connected to the combination of cells.
   Resistance of the resistor, R = 8.5Ω
   Current drawn from the supply=I is given by the relation,
   $I = \frac{nE}{R+nr}$
   $= \frac{6 \times 2}{8.5 + 6 \times 0.015} = \frac{12}{8.59} = 1.39\mathrm{A}$

Terminal voltage, V = IR = 1.39 $\times$ 8.5 = 11.87 A

Therefore, the current drawn from the supply is 1.39 A and terminal voltage is 11.87 A.

b. After a long use, emf of the secondary cell, E=1.9V

Internal resistance of the cell, r= 380 Ω

Maximum current

$E = \frac{E}{r} = \frac{1.9}{380} = 0.005\text{A}$

Therefore, the maximum current drawn from the cell is 0.005 A. This much current is not sufficient to start the motor of a car, as it requires very high current..

16: Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{\text{Al}} = 2.63 \times 10^{-8}\Omega\text{m}$**,** $\rho_{Cu} = 1.72 \times 10^{-8}\Omega\text{m}$ **Relative density of Al = 2.7, of Cu = 8.9.)**

Ans: Resistivity of aluminium, $\rho_{\text{Al}} = 2.63 \times 10^{-8}\Omega\text{m}$

Relative density of aluminium, d_1 = 2.7

Let l_1 be the length of aluminium wire and m_1 be its mass.

Resistance of the aluminium wire = R_1

Area of cross-section of the aluminium wire = A_1

Resistivity of copper, $\rho_{Cu} = 1.72 \times 10^{-8}\Omega\text{m}$

Relative density of copper, d_2 = 8.9

Let l_2 be the length of copper wire and m_2 be its mass.

Resistance of the copper wire = R_2

Area of cross-section of the copper wire = A_2

The two relations can be written as

$R_1 = \rho_1 \frac{l_1}{A_1}$ ...... (1)

$R_2 = \rho_2 \frac{l_2}{A_2}$ ...... (2)

It is given that,

$R_1 = R_2$

$\therefore \rho_1 \frac{l_1}{A_1} = \rho_2 \frac{l_2}{A_2}$

And,

$l_1 = l_2$

$\therefore \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2}$

$\frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}$

$= \frac{2.63 \times 10^{-8}}{1.72 \times 10^{-8}} = \frac{2.63}{1.72}$

Mass of the aluminium wire,

m_1 = Volume $\times$ Density

= $A_1 l_1 \times d_1 = A_1 l_1 d_1$ ... (3)

Mass of the copper wire,

m_2 = Volume $\times$ Density

= $A_2 l_2 \times d_2 = A_2 l_2 d_2$ ..... (4)

Dividing equation (3) by equation (4), we obtain

$\frac{m_1}{m_2} = \frac{A_1 l_1 d_1}{A_2 l_2 d_2}$

For $l_1 = l_2$,

$\frac{m_1}{m_2} = \frac{A_1 d_1}{A_2 d_2}$

For $\frac{A_1}{A_2} = \frac{2.63}{1.72}$

$\frac{m_1}{m_2} = \frac{2.63}{1.72} \times \frac{2.7}{8.9} = 0.46$

It can be inferred from this ratio that m_1 is less than m_2. Hence, aluminium is lighter than copper.

Since aluminium is lighter and its cost would be less and is preferred for overhead power cables over copper.

17: What conclusion can you draw from the following observations on a resistor made of alloy manganin?

CURRENT	VOLTAGE	CURRENT	VOLTAGE
0.2	3.94	3	59.2
0.4	7.87	4	78.8
0.6	11.8	5	98.6

0.8	15.7	6	118.5
1.0	19.7	7	138.2
2.0	39.7	8	158.0

Ans: It can be inferred from the given table that the ratio of voltage with current is a constant, which is equal to 19.7. Hence, manganin is an ohmic conductor i.e., the alloy obeys Ohm's law. According to Ohm's law, the ratio of voltage with current is the resistance of the conductor. Hence, the resistance of manganin is 19.7Ω.

18: Answer the following questions:

a. **A steady current flows in a metallic conductor of non uniform cross section of non uniform cross section. Which of these quantities is constant along the conductor,: current ,current density ,electric field, drift speed?**
b. **Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.**
c. **A low voltage supply from which one needs high currents must have very low resistance. Why?**
d. **A high tension (HT) supply of, say, 6 kV must have a very large internal resistance**

Ans:

a. When a steady current flows in a metallic conductor of non uniform cross-section, the current flowing through the conductor is constant. Current density, electric field, and drift speed are inversely proportional to the area of cross-section. Therefore, they are not constant in a conductor of varying cross section.
b. No, Ohm's law is not universally applicable for all conducting elements. Vacuum diode semi-conductor is a non-ohmic conductor. Ohm's law is not valid for it.
c. According to Ohm's law,
$I = \frac{V}{R}$
If V is low, then internal resistance R must be very low, so that high current can be drawn from the source.

d. In order to prohibit the current from exceeding the safety limit, a high tension supply must have a very large internal resistance. If the internal resistance is small, then the current drawn can exceed the safety limits in case of a short circuit which can be very dangerous.

19: Choose the correct alternative:

a. **Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.**
b. **Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.**
c. **The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.**
d. **The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of (10^{22}/10^{3}).**

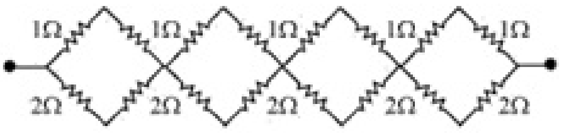
Ans:

a. Alloys of metals usually have greater resistivity than that of their constituent metals.
b. Alloys usually have lower temperature coefficients of resistance than pure metals.
c. The resistivity of the alloy, manganin, is nearly independent of increase of temperature.
d. The resistivity of a typical insulator is greater than that of a metal by a factor of the order of 10^{22}.

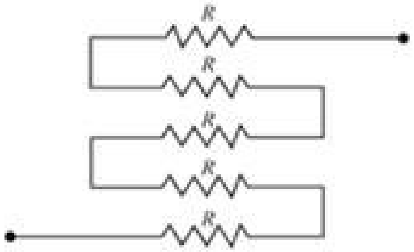
20:

a. **Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?**
b. **Given the resistances of $1\Omega, 2\Omega, 3\Omega,$ how will be combine them to get an equivalent resistance of**
(i) (11/3)Ω (ii) (11/5)Ω (iii) 6Ω (iv) (6/11)Ω?

c. **Determine the equivalent resistance of networks shown in fig.**



(a)



(b)

Ans:

a. Total number of resistors= n

   Resistance of each resistor= R

   i. When n resistors are connected in series, effective resistance R_1 is the maximum, given by the product nR.

      Hence, maximum resistance of the combination, R_1 = nR

   ii. When n resistors are connected in parallel, the effective resistance (R_2) is the minimum, given by the ration $\frac{R}{n}$.

      Hence, minimum resistance of the combination, $R_2 = \frac{R}{n}$

   iii. The ratio of the maximum to the minimum resistance is,

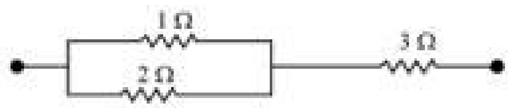
      $\frac{R_1}{R_2} = \frac{nR}{R/n} = n^2$

b. The resistance of the given resistors is, $R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 3\Omega$

   i. Equivalent resistance, $R' = \frac{11}{3}\Omega$

      Consider the following combination of the resistors.

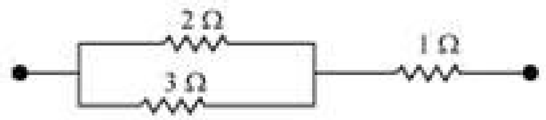
      Equivalent resistance of the circuit is given by,



      $R' = \frac{2\times1}{2+1} + 3 = \frac{2}{3} + 3 = \frac{11}{3}\Omega$

   ii. Equivalent resistance, $R' = \frac{11}{5}\Omega$

      Equivalent resistance of the circuit is given by,



$R' = \frac{2\times3}{2+3} + 1 = \frac{6}{5} + 1 = \frac{11}{5}\Omega$

iii. Equivalent resistance, $R' = 6\Omega$

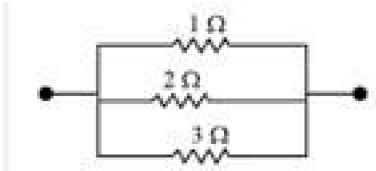
Consider the series combination of the resistors, as shown in the given circuit.

Equivalent resistance of the circuit is given by the sum,

$R' = 1 + 2 + 3 = 6\Omega$

Consider the series combination of the resistors, as shown in the given ciruit.

Equivalent resistance of the circuit is given by,



$R' = \frac{1\times2\times3}{1\times+2\times3+3\times1} = \frac{6}{11}\Omega$

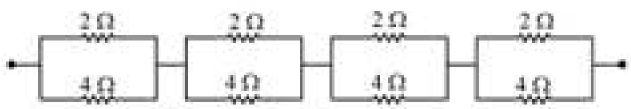
c. i. It can be observed form the given circuit that two resistors of resistance 1Ω each are connected in series.

Hence, their equivalent resistance= (1 + 1) = 2Ω

It can also be observed that two resistors of resistance 2Ω each are connected in series.

Hence, their equivalent resistance= (2 + 2) = 4Ω

Therefore, the circuit can be redrawn as



It can be observed that 2Ω and 4Ω resistors are connected in parallel in all the four loops.

Hence, equivalent resistance (R') of each loop is given by,$R' = \frac{2\times4}{2+4} = \frac{8}{6} = \frac{4}{3}\Omega$

The circuit reduces to

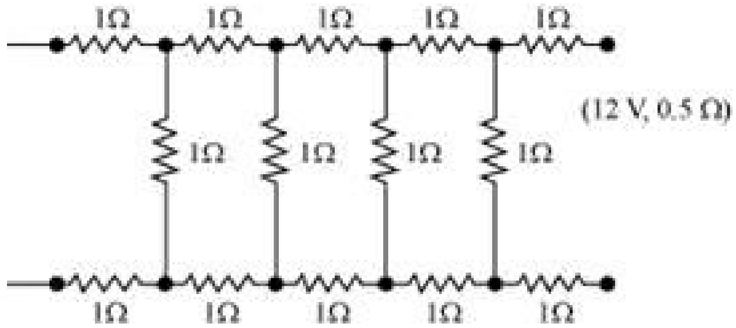
All the four resistors are connected in series.

Therefore, equivalent resistance of the given circuit is $\frac{4}{3} \times 4 = \frac{16}{3}\Omega$

ii. It can be observed from the given circuit that five resistors of resistance R each are connected in series.

Therefore, equivalent resistance of the circuit = R + R + R + R + R = 5R.

21: Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the infinite network shown in Fig 3.32. Each resistor has 1Ω resistance.



Ans: The resistance of each resistor connected in the given circuit, $R = 1\Omega$

Let the equivalent resistance of the given circuit = R'

The network is infinite, Hence, equivalent resistance is given by the relation,

$R' = 2 + \frac{R'}{R'+1}$

$R'^2 - 2R' - 2 = 0$

$R' = \frac{2 \pm \sqrt{4+8}}{2}$

$= \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$

Negative value of R' cannot be accepted. Hence, equivalent resistance,

$R' = (1 + \sqrt{3}) = 1 + 1.73 = 2.73\Omega$

Internal resistance of circuit is 0.5Ω

Therefore, total resistance of the given circuit $= 2.73 + 0.5 = 3.23\Omega$

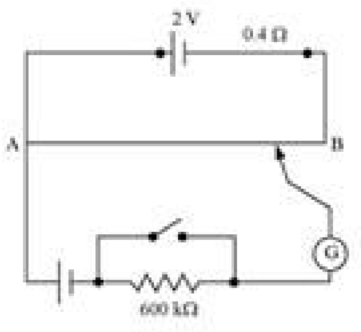
Supply Voltage, V = 12V

According to Ohm; s Law,

$I = \frac{12}{3.23} = 3.72\ A$

22: Figure shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents up to a few mA) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in

series with it, which is shorted close to the balance point. The standard cell is then replace by a cell of unknown emf and the balance point found similarly, turns out to be at 82.3 cm length of the wire.



a. **What is the value ε ?**
b. **What purpose does the high resistance of 600 $k\Omega$ have?**
c. **Is the balance point affected by this high resistance?**
d. **Is the balance point affected by the internal resistance of the driver cell?**
e. **Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?**
f. **Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?**

Ans:

a. Constant emf of the given standard cell, E_1 = 1.02V

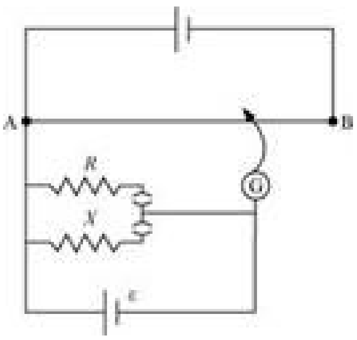
   Balance point on the wire, l_1 = 67.3cm

   A cell of unknown emf, ε , replaced the standard cell. Therefore, new balance on the wire, l = 82.3cm

   The relation connecting emf and balance point is,
   $\frac{E_1}{l_1} = \frac{\varepsilon}{l}$
   $\varepsilon = \frac{l}{l_1} \times E_1$
   $= \frac{82.3}{67.3} \times 1.02 = 1.247V$
   The value of unknown emf is 1.247V.
b. The purpose of using the high resistance of 600$k\Omega$ is to reduce the current passing through the galvanometer when the movable contact is far from the balance point.

c. The balance point is not affected due to the presence of high resistance.
d. The point is not affected by the internal resistance of the driver cell.
e. The method would not work if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V. This is because if the emf of the driver cell of the potentiometer is less than the emf of the other cell, then there would be no balance point on the wire as potential developed in the potentiometre wire will always be less than the emf of cell.
f. The circuit would not work well for determining an extremely small emf. As the circuit would be unstable, the balance point would be close to end A. Hence, there would be a large percentage of error.
Modification: The given circuit can be modified if a series resistance is connected with the wire AB. The potential drop across AB is slightly greater than the emf measured. The percentage error would be small.

23: Figure shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor R = 10.0Ω is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What might you do if you failed to find a balance point with the given cell of emf ε?



Ans: Resistance of the standard resistor, R = 10.0Ω

Balance point for this resistance, l_1 = 58.3 cm

Current in the potentiometer wire = i

Hence, potential drop across R, E_1 = iR

Resistance of the unknown resistor = X

Balance point for this resistor, l_2 = 68.5 cm

Hence, potential drop across X, E_2 = iX

The relation connecting emf and balance point is,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

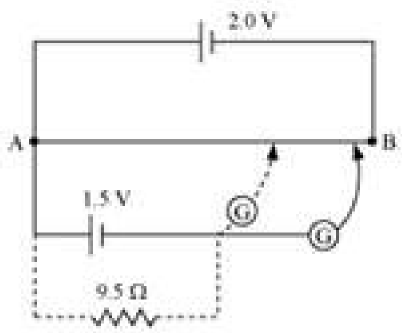
$\frac{iR}{iX} = \frac{l_1}{l_2}$
$X = \frac{l_1}{l_2} \times R$
$= \frac{68.5}{58.3} \times 10 = 11.749\Omega$
Therefore, the value of the unknown resistance, X, is 11.75Ω.

If we fail to find a balance point with the given cell of emf , ε, then the potential drop across R and X must be reduced by inserting a high resistance in series with it. Only if the potential drop across R or X is smaller than the potential drop across the potentiometer wire AB, a balance point is obtained.

24: Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.

Ans: Internal resistance of the cell=r

Balance point of the cell in open circuit, l_1 = 76.3cm



An external resistance (R) is connected to the circuit with $R = 9.5\Omega$

New balance point of the circuit, l_2 = 64.8cm

Current flowing through the circuit= I

The relation connecting internal resistance of cell and emf is given by,

$r = \frac{(l_1 - l_2)}{l_2} \times R$
$= \frac{76.3 - 64.8}{64.8} \times 9.5 = 1.68\Omega$
Therefore, the internal resistance of the cell is 1.68Ω.